Inter-harmonic parameters estimation in power grid based on accelerated PSO and $T_{\frac{5}{11}}$ window

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Abstract: A novel method on inter-harmonic parameters estimation is proposed. It contains two steps: pre-calculation and solution optimisation. An interpolated-discrete fourier transform (DFT)-based algorithm with a fifth-order-triangular and eleventh-order-rectangular ($T_{\frac{5}{11}}$) window is applied in the pre-calculation process. Owing to mainlobe interferences caused by inter-harmonics in the frequency domain, the accelerated particle swarm optimisation (PSO) algorithm is proposed to optimise the solution of the pre-calculation. Three aspects in the PSO are accelerated including utilising the especial DFT expression of $T_{\frac{5}{11}}$ window, applying a changing inertia weight and locking the frequency. From the view of estimating accuracy, the errors caused by spectral leakage can be restrained to a degree of $10^{-11}$ by using $T_{\frac{5}{11}}$ window, and the relative errors of parameters could be optimised to a degree of $10^{-5}$ at the tenth generation in the accelerated PSO. More simulations show that the proposed method can address many complex situations caused by inter-harmonics, and corresponding detection accuracies satisfy the requirements of national and international standards.

1 Introduction

Harmonics detection and parameters estimation are both important tasks in power quality research [1–3]. Many types of researches [4–9] dealing with harmonic problems mainly focus on methods based on windowed and interpolated DFT (WiDFT-based or IpDFT-based method). Commonly, they contain a similar structure.

First, use a generalised cosine-window function and DFT to transform the electronic signal from the time domain to the frequency domain (to restrain spectral leakage). The maximum sidelobe decay (MSD) window, also known as the I-class Rife-Vincent window, is used commonly. Second, construct an interpolation equation based on the spectra in the frequency domain (to address the picket-fence effect). Finally, estimate the parameters by solving the root of this equation.

Actually, it has to be acknowledged that these approaches work effectively in harmonic detection and parameters estimation. A kind of MSD window with eight coefficients was introduced in [4, 5] (finally, a window with two coefficients was chosen). In a 50 dB environment, the relative errors (RE) of amplitudes (RAE) and phases (RPE) reach to about $10^{-2}$ and $10^{-4}$ respectively [4], and the relative frequency error (RFE) reach to about $10^{-3}$ [5]. After that, the Cramer–Rao bound [10] is analysed as well. When a phase jump happens, the voltage can be adjusted to the new level in about 20 ms. By using a digital signal processor (DSP) with a TMS320C6713 processor, the parameters can be estimated in real-time in practical applications. In the literature [6], a specific MSD window with a three-point interpolation method was adopted, and the variance of frequency reaches to about $10^{-4}$ in a 20 dB environment. The algorithm in [6] mainly aimed at the frequency measurement. It is noteworthy that many powerful IpDFT-based algorithms in other papers [7–9] are analysed and compared in [6], hence a comprehensive view of the IpDFT-based routine can be obtained according to the analysis and conclusion.

However, as the fast developing of the distribution network, a huge number of non-linear elements are applied into the power grid. Consequently, inter-harmonics [11, 12], whose frequencies are not the multiple of the fundamental frequency, are likely to appear. Big trouble caused by inter-harmonics is the mainlobe interference (MLI). Owing to irregular frequency, the mainlobes of two sinusoidal waves are likely to overlap with each other. This phenomenon is harmful to IpDFT-based methods because the spectra selected to contribute the interpolation formula should be as precise as possible.

To address this question, atomic decomposition (AD), was used in paper [13]. The overlapped area in the frequency domain is separated apart according to the basis of AD. Then, parameters of sinusoids could be estimated separately. Wavelet transform (WT) is also a kind of similar method [14, 15]. It contains different bases from the AD. For example, the Daubechies-20 basis was applied in [14], and a multi-alternative basis (including a finite-impulse response and an infinite-impulse response) was used in [15] to decompose the time-domain signal. The corresponding results show that these methods could lead to feasible accuracies to some degree, e.g. the RAE acquired in [13] is about 3.1%. However, the simulations also show that neither WT nor AD could decompose the information in the frequency domain fully, because the shape of a mainlobe does not perfectly match the basis of either. Considering this, it might be more reasonable to reconstruct the spectra based on the parameters which have been estimated. Then, the group of parameters can be optimised based on the difference between the spectra reconstructed and the practical spectra.

With the analysis above, a revised parameters estimation method is proposed in this paper, aiming at addressing the MLI caused by inter-harmonics. It contains two steps: the first step, named as pre-calculation, is to acquire an approximate solution via IpDFT, which is similar to a common IpDFT but using the specific $T_{\frac{5}{11}}$ window. The second step is to optimise this solution by using accelerated particle swarm optimisation (PSO).

A challenge is that an extra step (PSO) is added after IpDFT, which might result in more time-consuming. Hence, the primary task is to improve the algorithm and rise the calculating speed. An advantageous aspect is that the solutions acquired via IpDFT can be used to initialise the particles in PSO. It saves much time on widely searching. Thus, the proposed PSO is not overmuch time-consuming. In addition, some other aspects of the proposed PSO...
are also revised, and they are explained detailedly in the main body of this paper.

However, the strategies above are not enough. According to the requirement in standard IEC 61727–2004 [16], the time for feeding back harmonic parameters should be compressed in 2.5 periods of a 50 Hz grid signal, which means a common PSO algorithm needs to be accelerated largely to ensure the time on iterating is no more than 50 ms.

To satisfy this requirement, a specific $T_{11}R_{11}$ window is applied: a DFT based on a generalised cosine window would produce at least three vectors in the frequency domain, whereas the DFT based on a triangular (Tria.) window or a rectangular (Rect.) window only produces one. Although it slightly effects the time consumed in the IpDFT process, it accelerates the speed in iterating by over three times, because fewer vectors are used to reconstruct the spectra. This is large merit of using a $T_{11}R_{11}$ window rather than an MSD window.

However, due to not using generalised cosine windows, another question also needs to be considered: the sidelobe value of a single Tria. window or Rect. window is higher than the one of an MSD window, which is not helpful to get a precise solution. Therefore, Tria. units and Rect. units need to be convolved to overcome this weakness.

Overall, the structure of this paper is clarified: the whole pre-calculation process is shown in Section 2. In this part, a $T_{11}R_{11}$ window is constructed. The capability of this window on addressing common harmonic questions, i.e. spectral leakage and picket-fence effect, is demonstrated as well. Then, the particles in PSO can be initialised according to the solution obtained via IpDFT. In Section 3, the accelerated PSO is applied to optimise the initial particles. Situations which might be met in practical detections are simulated in Section 4. Then, a practical case is given to confirm the feasibility of this method in Section 5. Finally, conclusions are drawn in Section 6.

## 2 Pre-calculation based on a $T_{11}R_{11}$ window

The objective of this section is to pre-calculate a solution by using IpDFT with a $T_{11}R_{11}$ window. According to the accounts in Section 1, the $T_{11}R_{11}$ window is designed specially. Hence, the details about how to construct it are arranged in Section 2.1. After that, a two-point interpolation formula is applied to estimate the parameters of inter-harmonics or harmonics, which is shown in Section 2.2.

### 2.1 To construct a $T_{11}R_{11}$ window

The first step of detection is to acquire the discrete signal from the power grid. A kind of pincer-shaped Rogowski coil [17] was used, and it served as the sensor. After transducing the current/voltage to the secondary side, a discrete signal can be sampled via electric sampling instruments. All the practical instruments are shown in Fig. 1.

Theoretically, the sampled discrete signal can be regarded as an $N$-length signal $x(n)$ which satisfies the equation below:

$$x(n) = \sum_{z=1}^{Z} A_z \sin(2\pi f_n T_z + \phi_z)$$

where $T_z$ is the sampling interval, with $T_z = 1/F_z$. The sampling frequency $F_z$ is related to the practical sampling instrument, thus $T_z$ and $F_z$ are both known; $n$, with $n = 0, 1, \ldots, N - 1$, is the nth point of samples; $z$ denotes the zth sinusoid; and $Z$ is the number of total sinusoids; $f_n, A_z$ and $\phi_z$ are the frequency, the amplitude and the phase of the zth sinusoid, respectively.

To cooperate with $x(n)$, the length of the $T_{11}R_{11}$ window should be equal to $N$, the same as the length of $x(n)$. This window is made of 16 length-equal units, and the length of each unit, marked as $M$, is 1/16 of $N$. Among all these units, there are 5 Tria. units and 11 Rect. units.

The expression of a Tria. unit is given in (2), and the corresponding DFT is given in (3):

$$w_{T}(m) = 1 - \frac{2m - (M - 1)}{M}$$

$$W_{T}(k) = 2 \sin^2(k\pi/2) e^{-j\pi M - j\pi M}$$

where $m$, with $m = 0, 1, \ldots, M - 1$, denotes the mth point in the Tria. unit and $k$ denotes the k-Hz point in the frequency domain.

Note that the expressions of an odd-length Tria. unit are different from the ones in (2) and (3). Then, the $T_{11}R_{11}$ window, which is made of four fourth-order windows, can be obtained as below:

$$w_n(n) = (w_{TR} w_{RR} w_{RR}) (w_{TR} w_{RR} w_{RR})$$
Different triangular-to-rectangle ratios (T–R ratio) would result in different 16-order windows. To compare their capabilities, their frequency-domain spectra (transformed via DFT) are all shown in Fig. 2. Note the frequency $k$ is normalised here, marked as $\beta$ in (9). This normalisation makes it easy to compare the proposed window with other windows since the interference caused by length has been eliminated.

$$\beta = 2\pi k/M$$  \hspace{1cm} (9)

For the number of Tria. units (T-number) and the number of Rect. units (R-number), their summation is 16. It is found in simulations that if the T-number increases, the mainlobe becomes thinner but the maximum sidelobe goes higher. The R-number works on the contrary. Hence, this factor can be used to adjust the capability of the final 16-order window.

Among all these windows, the sidelobe value of window corresponding to 5T:11R is low enough, about $-226$ dB, which would cause errors to the harmonic estimators at a degree of $10^{-11}$. It means the errors caused by spectral leakage are restrained well. In addition, mark the point whose loudness is the same as the max sidelobe, i.e. $-226$ dB, on the mainlobe. The normalised frequency of this point is about 1.085 pu. This value is relatively low compared with the values corresponding to other T–R ratios, which means the width of the mainlobe is not so large. Overall, the ratio 5T:11R balances the impact from the sidelobes and the impact range of the mainlobe. Hence, it might be more suitable than others.

2.2 To acquire estimators via IpDFT

After acquiring the $T_{3}R_{11}$ window, multiply it with the signal $x(n)$, in (1), and the windowed signal $x_{w}$ can be obtained, as (10). The corresponding DFT is given in (11).

$$x_{w}(n) = x(n) w(n)$$ \hspace{1cm} (10)

$$X(k) = \sum_{n=0}^{N-1} x_{w}(n) e^{-j2\pi kn/M} = \sum_{\ell=1}^{Z} A_{\ell} e^{j\phi_{\ell}} W(k^{'})$$ \hspace{1cm} (11)

where $k$ denotes the $k$-Hz point in the frequency domain. $W(k^{'})$ denotes the revised DFT of the $T_{3}R_{11}$ window. Note that the DFT formula is revised here. For a common DFT, the denominator in the exponential function shall be $N$, while it is changed to $M$, i.e. $1/16$ of $N$. This adjustment is necessary because only in this way DFT of the $z$th sinusoid could be deduced from (11) and (12) (see (12)), where $k'$ is the adjusted frequency corresponding to $k$, and it can be calculated via the equation below:

$$k' = k - f_s M/F_s$$ \hspace{1cm} (13)

Equation (12) can be comprehended in the meaning of mathematics: DFT of the windowed signal produces a vector at the $k$-Hz point. Its amplitude and phase are marked as $L(k')$ and $\theta(k')$, and given in (14) and (15), respectively

$$L(k') = \frac{16 A_{1} \sin^{11}(k' \pi) \sin^{10}(k' \pi/2)}{M \sin^{10}(k' \pi/M) \tan(k' \pi/M)}$$ \hspace{1cm} (14)

$$\theta(k') = \phi_{\ell} - \pi/2 - 16k'\pi(M - 1)/M$$ \hspace{1cm} (15)

To construct a two-point interpolation equation, two spectra with the largest values in the local frequency domain, i.e. $|X(k_1)|$ and $|X(k_2)|$, are picked out. Their frequencies are marked as $k_1$ and $k_2$ from left to right. These four variables are all known. Suppose the variable $\delta$ is the distance between $k_1$ and the theoretical frequency point, i.e. $k_1' = f_s M/F_s$. Then, (16) is available

$$-\delta = k_1 - k_2 = k_1'$$ \hspace{1cm} (16)

where $d$ is the distance between two adjacent spectra in the frequency domain. It depends on the frequency resolution, which is relative to practical instruments (known). Mark $\alpha$ as the ratio of $|X(k_1)|$ to $|X(k_2)|$, then (17) is available

$$X(k) \text{for the } z \text{th sinusoid} \approx A_{\ell} e^{j\phi_{\ell}} W(k')^{16} \sin^{10}(k' \pi/2) e^{j(k_1 - k_2 - 16k'\pi(M - 1)/M)}$$ \hspace{1cm} (12)

$$\pm L(k') \angle \theta(k')$$

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**Fig. 2** Frequency-domain information of window corresponding to each T–R ratio
It searches a more precise solution with the guidance of the accuracy requirement and the real-time-capability requirement.

For the spectrum $X(k_i)$, it can be expanded as (19). Combined with (16) and $\delta$, the estimators of parameters versus the $i$th sinusoid can be deduced and given in (20)

$$X(k_i) = \frac{A_i}{2j} \exp(j\varphi_i) W_1^\prime(-\delta) W_R^\prime(-\delta)$$

(19)

$$\begin{align*}
\hat{f}_i &= (k_i + |A_i|)5/\deltaM \\
\hat{A}_i &= 2X(k_i)W_1^\prime(\delta) W_R^\prime(\delta) \\
\hat{\varphi}_i &= \hat{\varphi}_i(-\delta) + \pi2 + 16\delta(\deltaM - 1)/\deltaM \\
\end{align*}$$

(20)

where $\hat{A}_i$, $\hat{f}_i$ and $\hat{\varphi}_i$ are estimators of the amplitude, the frequency and the phase, respectively.

In addition, some tips are also found during simulating: the two spectra with the maximum values in the local frequency domain are suggested to be chosen when MLI does not exist. It helps to restrain the influence caused by sidelobes of the fundamental wave. However, if MLI exists, it is suggested to choose the spectra a little far away from the peak. However, the loudness of the chosen spectra should not be more than $-160$ dB; otherwise, the restraining effort to spectral leakage would not be enough.

If no inter-harmonic appears, the estimators calculated via (20) contain a pretty high accuracy, and the outline of spectra in the local frequency domain seems like a hill. However, inter-harmonics may cause MLI in the local frequency domain, and it damages both the estimating accuracy and the shape of the hill, shown in Fig. 3. For example, by using the methods in [5, 6], the harmonics may cause MLI in the local frequency domain, and it interferes with each other and the purple spectra are the composing results.

Fig. 3 Distortion of the spectra impacted by inter-harmonic; the blue hill is the mainlobe of a non-MLI sinusoid. The red hill and the green hill interfere with each other and the purple spectra are the composing results.

3 Optimise the solution via accelerated PSO

3.1 Optimise solution via PSO

It is hard to obtain the precise parameters via IpDFT purely because MLI influences the spectra in the local frequency domain. Hence, PSO [19, 20] is employed to optimise the approximate solution. It searches a more precise solution with the guidance of fitness, instead of decomposing the spectra in the frequency domain. Moreover, PSO runs faster than algorithms such as a genetic algorithm and simulated annealing, because it contains a simpler iterating structure. These advantages make it satisfy both the accuracy requirement and the real-time-capability requirement.

The work flow of the accelerated PSO is shown in Fig. 4. Although the structure is almost the same with a common PSO, detailed calculations in some steps are improved, which are highlighted with a yellow background in Fig. 4. Several details which are of importance for constructing PSO are specified as follows.

Fig. 4 Work flow of the accelerated PSO algorithm.

3.1.1 To initialise every particle: The $i$th particle in the whole swarm is marked as $p(i)$, which is constituted as (21). Every particle should contain all parameters of sinusoids which cause the MLI in the local frequency domain. The matrix $f$, the matrix $A$ and the matrix $\varphi$ in (21) are given in (22)–(24), respectively

$$p(i) = [f_i, A_i, \varphi_i]$$

(21)

$$f = [f_1, f_2, ..., f_i]$$

(22)

$$A = [\hat{A}_1, \hat{A}_2, ..., \hat{A}_i]$$

(23)

$$\varphi = [\hat{\varphi}_1, \hat{\varphi}_2, ..., \hat{\varphi}_i]$$

(24)

where $z'$ represents the $z$'th sinusoid that appears in the local frequency domain. $f_i$, $\hat{A}_i$ and $\hat{\varphi}_i$ are its frequency, amplitude and phase, respectively. These variables are initialised according to the estimators obtained via (20), and they will be updated with the iteration number growing.

3.1.2 To calculate the fitness: For the parameters in particle $p(i)$, if they are precise enough, the spectra reconstructed based on them should be close to being spectra obtained via DFT. Hence, the fitness can be considered as the degree of deviation between the spectra reconstructed, marked as $Q_L$, and the practical spectra obtained via DFT, marked as $Q_Z$. The more similar they are, the better the fitness is.

The spectra $[\text{vectors } X(k_i)]$ obtained via DFT have been calculated in Section 2, and a group of them $Q_Z$ in the local frequency domain hill can be picked out for observation. They are given in (25). $G$ is the length of this group

$$Q_Z = [X(k_1'), X(k_2'), ..., X(k_G')]$$

(25)
For the gth element in $Q_g$, $k'_g$ is the corresponding frequency. Then, all the frequencies can be arranged in ascending order, which is given in the equation below:

$$f_g = [k'_1, k'_2, \ldots, k'_g]^T$$  \hspace{1cm} (26)

To reconstruct $\hat{Q}_g$, parameters in $p(t)$ should be used: let each element in $f$ serves as the variable $k$ in (13), and element in $f_g$ serves as the variable $f_g$ in (13). Then, a $G$-row and $z'$-column matrix $D$ can be obtained, as (27). Each element $D_{g,z}$ in this matrix is calculated via (28)

$$D = \begin{bmatrix} D_{1,1} & D_{1,2} & \cdots & D_{1,z} \\ \vdots & \ddots & \ddots & \vdots \\ D_{G,1} & \cdots & D_{G,z} \end{bmatrix} \hspace{1cm} (27)$$

It can be observed that the element in (28) contains the same meaning as $k'_g$ in (13). Thus, the spectra $Q$ can be reconstructed by substituting $D_{g,z}$ for the variable $k'_g$ in (14) and (15). Then the matrix of results marked as $\hat{Q}$ is given in (29). In the meaning of mathematics, the vector at the $g$th row and the $k'_g$th column in this matrix denotes the spectrum constructed by the $z$th sinusoid at the $k'_g$-Hz point

$$\hat{Q} = L \exp(j \theta) \hspace{1cm} (29)$$

$$L = L_g(D) \hspace{1cm} (30)$$

$$\theta = \theta_g(D) \hspace{1cm} (31)$$

To analyse the vectors in $\hat{Q}$ further, the amplitude and the phase of the vector are arranged in (30) and (31), marked as $L$ and $\theta_g$, respectively. These three matrices are all $G$-row and $z'$-column.

Sum the elements in $\hat{Q}$ by rows and the final reconstructed spectra set $\hat{Q}_g$ can be obtained

$$\hat{Q}_g(z) = \sum_{g} \hat{Q}_{g,z} \hspace{1cm} (32)$$

According to the previous description, fitness represents the degree of deviation between $\hat{Q}_g$ and $Q_g$. All elements in these matrices are vectors; therefore, the deviation can be exhibited in two aspects, i.e. amplitude and phase of the vector. Hence, fitness can be designed and given in the equation below:

$$\text{min} \ \text{Fit}(i) = \frac{1}{2} \times \left[ \left( \frac{1}{G} \sum_{g} \left| X(k'_g) - \hat{Q}_g(z) \right| \right) \right] + \arg \left( \frac{1}{G} \sum_{g} \frac{\arg(X(k'_g)) - \arg(\hat{Q}_g(z))}{\arg(X(k'_g))} \right) \hspace{1cm} (33)$$

where symbol $\arg(\cdot)$ denotes the phase of the inside vector; $\text{Fit}(i)$ is the fitness of $p(i)$; $X(k'_g)$ is the $g$th element in $Q_g$; and $\hat{Q}_g(z)$ is the $g$th element in $\hat{Q}_g$.

Equation (33) describes the fitness as the summation of RaE and RpE of all spectra. Therefore, the less the summation is, the better the fitness is.

3.1.3 To update new particles: After calculating the fitness corresponding to every particle, all the fitness values will be compared with the best-known position.

For the $i$th particle: its best-known position $b(i)$ is documented from the first generation. It would be updated only if the fitness $p(i)$ is better than the fitness of $b(i)$. This updating expression is given in the equation below:

$$b(i) \leftarrow p(i), \text{ if } p(i) \text{ is better than } b(i) \hspace{1cm} (34)$$

For the whole swarm: $p_{\text{best}}$ is the best-known position among all the particles and $p_{\text{best}}$ would be updated only if the fitness of $p(i)$ is better than the fitness of $p_{\text{best}}$, as below:

$$p_{\text{best}} \leftarrow p(i), \text{ if } p(i) \text{ is better than } p_{\text{best}} \hspace{1cm} (35)$$

During updating new particles, the relation between velocity $v(i)$ and other variables above is given in (36). After that, the new particle $p(i)$ could be calculated as (37)

$$v(i) = c_1 \times \text{rand}([b(i) - p(i)]) + c_2 \times \text{rand}([p_{\text{best}} - p(i)]) + \omega \times v(i) \hspace{1cm} (36)$$

$$p(i) = p(i) + v(i) \hspace{1cm} (37)$$

where $\omega$ is the inertia weight; $p(i)$ is the $i$th particle (previous); $p(i)$ is the $i$th particle after updating; $v(i)$ is the velocity of $p(i)$; $v(i)$ is the velocity of $p(i)$; $c_1$ and $c_2$ are acceleration constants (known).

The symbol $\text{rand}$ denotes a random real in the interval $[0, 1]$.

3.2 Approaches to accelerating the PSO

A large number of particles are needed in PSO to search for the best solution, which would make the method not efficient. Thus, the proposed PSO should be accelerated in three aspects, to strengthen the real-time-detection capability.

3.2.1 Utilise the advantage of $T_{q}R_{11}$ window: Commonly, high-order MSD windows perform better in analysing spectral information than a Tria. or a Rect. window does because they contain lower sidelobes and a larger mainlobe-reducing gradient, which is helpful to restrain spectral leakage. The universal DFT formula of an MSD window is given in the equation below:

$$W_{\text{MSD}}(k) = \sum_{q} a_q \frac{1}{2}[W_R(k + q) + W_R(k - q)] \hspace{1cm} (38)$$

where $a_q$ is the $q$th coefficient of this window and $W_R(k)$ is given in (5). Both the range of $q$ and the value of $a_q$ depending on the type of the window. For $q = 0, 1$, it is equivalent to the Hann window.

Finally, $T_{q}R_{11}$ window is employed after comparing (38) with (3) and (5), because a triangular window or a rectangular window only produces one vector at a point in the frequency domain. However, even a Hann window, the simplest one in the cosine-window family (except Rect.), will produce three vectors, i.e. $W_R(k)$, $W_R(k + 1)$ and $W_R(k - 1)$. These vectors will aggravate the amount of computing from (28) to (31). Moreover, this amount will be amplified obviously by the iterations in PSO and by the number of particles. Therefore, by using $T_{q}R_{11}$ window, much time can be saved in practical detection.

3.2.2 Use a changing inertia weight instead of a constant weight: In some traditional PSO algorithms [19, 20], the inertia weight is (36) is a constant. This selection is not helpful to reach the best solution fast. The velocity of every particle is suggested to be large in the early iterations. It helps the swarm to optimise particles with a holistic view and avoids getting into a local-optimum solution. Subsequently, if a relatively appropriate optimising orientation has been obtained, the velocity is suggested to be less, in order to avoid wasting time on searching other directions.
Where standard, the bias of frequency estimator should be controlled of 10^{-8}. Optimising it via PSO is not so much. On the basis of this consideration, it is suggested to lock the frequency with a threshold. Thus, this configuration is conservative but reliable.

### 3.2.3 Lock the frequency

For the three sinusoidal parameters to be optimised, i.e. frequency, amplitude and phase, the changing tendency of the inertia weight along with iterations can be shown in Fig. 5. It can be observed that the degree of the weight stays at a high level (about 0.9) at the beginning, and it decreases sharply when iterations number approaches to the end point. This strategy pays more attention to avoid getting into a local-optimum solution.

With the analysis above, a kind of conservative decaying weight is applied in this work

\[ \omega = \omega_s - (\omega_s - \omega_wd) \times (t/T_m)^\frac{1}{3} \]  

(39)

where \( \omega_s \) is the starting weight; \( \omega_wd \) is the ending weight; \( t \) is the iteration number; and \( T_m \) is the maximum.

A suggested configuration is given in the following: the starting weight \( \omega_s \) is equal to 0.9 and \( \omega_wd \) is equal to 0.4. Then, the decay tendency of the inertia weight along with iterations can be shown in Fig. 5. It can be observed that the degree of the weight stays at a high level (about 0.9) at the beginning, and it decreases sharply when iterations number approaches to the end point. This strategy pays more attention to avoid getting into a local-optimum solution. Thus, this configuration is conservative but reliable.

### Table 2

<table>
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<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
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<td>( N_p )</td>
<td>number of particles</td>
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<td>( t_f )</td>
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<td>( t_L )</td>
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<td>( G )</td>
<td>frequency-domain window length</td>
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<td>( N_m )</td>
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<tr>
<td>( t_u )</td>
<td>executing time of CPU</td>
<td>( 2 \times 10^{-9} ) s</td>
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Last column includes example values.

### Table 3

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<th>Pha., deg</th>
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<tr>
<td>harm-7</td>
<td>350</td>
<td>23</td>
<td>105</td>
</tr>
<tr>
<td>inter-4</td>
<td>360</td>
<td>4</td>
<td>165</td>
</tr>
<tr>
<td>harm-9</td>
<td>450</td>
<td>37.6</td>
<td>33</td>
</tr>
</tbody>
</table>

\(^a\)Inter-n is the abbreviation of the nth inter-harmonic.

\(^b\)Harm-n is the abbreviation of the nth harmonic.

### 4 Simulation cases

#### 4.1 Conventional inter-harmonic detection

All simulations are executed in the same detection system. The sampling frequency is 5000 Hz, and the length of the signal is 1696 point, which is irregular enough to induce spectral leakages and picket-fence effect. The signal to be detected, similar to (1), is mixed by an array of sinusoids including the fundamental wave, harmonics and inter-harmonics. Parameters corresponding to each branch of sinusoids refer to the data in paper [13], and they are listed in Table 3.

Considering each basic calculation time might not cost the same time. The summation is increased by a factor of 3 (as margin), to ensure the practical time consumed would not surpass the requirement (50 ms) of IEC 61727–2004. On the basis of the values in Table 1, the time needed for total iterations in PSO can be approximated, as below:

\[ t_{PSO} \approx 6N_p\hat{t} \approx (20t_f - I_L)G\omega_0 + 6t_f G + 4(3t_f - I_L)N_m \]  

(40)

where \( \hat{t}_{PSO} \) is the time estimated for a whole PSO. Other variables in (40) are listed in Table 2. To give a more intuitive comprehension, an array of example values is added in Table 2. With these values, the time consumed is estimated to be about 35.49 ms. Besides, the time cost in IpDFT process is about 0.5 ms. Overall, the aggregate time of the method generally satisfies the requirement of standards.

#### 4.2 Determination of local frequency

After transforming the signal with the \( T_{fs} \) window and the revised DFT, the spectra in the frequency domain can be obtained, and shown in Fig. 6. It can be inferred from Fig. 6: several inter-harmonics exist in the detected signal because the first and the fifth local hills are disturbed by MLI obviously. Thus, the spectra whose loudness...
degrees are about $-120$ dB (at both the left-hand side and the right-hand side) are selected to construct the two-point interpolation equation, according to (17). For other hills not interfered, the interpolation formula is constructed based on the highest two spectra.

After pre-calculation, the parameters of sinusoids impacted by MLI are picked out as initial values in PSO, which are given in (41) and (42), respectively.

$$
p_{i+1} = \left[ \begin{array}{c} 49.8 \\ 56.47 \\ 350 \\ 360 \end{array} \right] \sin \left( \frac{2\pi}{nT} + 132^\circ \right) \quad (41)
$$

$$
p_{i+1} = \left[ \begin{array}{c} 379.8 \\ 23 \\ 23 \\ 4002 \\ 165 \end{array} \right] \sin \left( \frac{2\pi}{nT} + 43^\circ \right) \quad (42)
$$

where $p_{i+1}(i)$ is corresponding to the two sinusoids at the hill in Fig. 6b. $p_{i+1}(i)$ is corresponding to the ones at the hill in Fig. 6c. Then, these particles will be updated based on the configuration given in Table 4.

The practical computing efficiency depends on the practical instruments. According to the analysis in Section 3.2, the time cost for per iteration is about $3.37 \times 10^{-5}$ s, using a 500 MHz DSP. Considering the time-consuming is changeable, it might be more reasonable to analyse this algorithm through iteration numbers rather than computing time.

The trend of fitness along with iteration number is given in Fig. 7. The $y$-axis denotes the fitness of $p_{\text{best}}$, which reaches to a degree $10^{-5}$ at the tenth generation. Besides, the RE of parameters is of importance as well. Thus, the average values of $R\alpha E$, $R\gamma E$ and $R\phi E$ (AvRE) are given in the following: the AvRE corresponding to the tenth generation is about $1.68 \times 10^{-5}$ pu, which fully satisfies the requirement of the power grid.

The methods in some state-of-the-art papers [4–6] are simulated with the same parameters. All the results are arranged in Table 5 as contrasts. It can be observed: in a non-MLI situation, all the accuracies of estimators (obtained via methods in [4–6] and this paper) are much higher than the requirements of standards. However, for sinusoids impacted by MLI, REs obtained via the proposed method are much lower than the ones in the other two papers. A similar phenomenon also appears in the environment with 50 dB noise.

Besides, it is noteworthy that there is an abnormal phenomenon: for the parameters impacted by MLI with 50 dB noise, their accuracies are even better (lower values) than the ones not impacted by MLI. The reason is: although the sinusoids are impacted by MLI, the extra PSO process could optimise the RE of parameters to a very low degree. Moreover, this degree is even less than the RE obtained in a common detection with no MLI. Thus, this phenomenon confirms the accelerated PSO is indeed necessary to improve the accuracy.

### 4.2 Detection with continuous MLI

Unlike some methods trying to decompose the signal in the frequency domain or in the time domain, the accelerated PSO reconstructs spectra based on the solution of pre-calculation and then optimises it. There is an advantage for doing this: when continuous MLI exists in the frequency domain, it is still not difficult to reconstruct spectra. As a contrast, methods based on decomposing might be much more complicated and inaccurate. To verify this function, a signal with continuous MLI is created and detected. The parameters of this signal are given in the equation below:

$$
x(n) = 72 \sin(2\pi \times 62.32 \times nT_s + 132^\circ) + 98 \sin(2\pi \times 76.75 \times nT_s + 122^\circ) + 80 \sin(2\pi \times 97.1 \times nT_s + 43^\circ) + 92 \sin(2\pi \times 112.5 \times nT_s + 76^\circ) \quad (43)
$$

Multiply (43) with $T_2 R_{11}$ window, and transform the product via (11). Then, the spectra in the frequency domain (practical DFT) can be obtained and shown in Fig. 8 in the red line. Meanwhile, for every single sinusoid, multiply it with the same $T_2 R_{11}$ window, and compute the DFT of the product. Then, the theoretical spectra of each of them can be acquired and shown in Fig. 8 as well.

It can be observed: the difference degree between the spectra computed from (43) and the ones corresponding to every single sinusoid is very large. It implicates MLI impacts this area largely.

Then, the solution of pre-calculation can be obtained by constructing interpolation formula with the spectra in Fig. 8. These parameters are arranged in (44), as initial values in PSO and the corresponding REs are listed in Table 6.

**Table 4** Configuration of iteration parameters in PSO

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight $\omega_{\text{fL}}$</td>
<td>0.9</td>
<td>weight $\omega_{\text{fL}}$</td>
<td>0.4</td>
</tr>
<tr>
<td>constant $c_1$</td>
<td>1.2</td>
<td>constant $c_2$</td>
<td>1.3</td>
</tr>
<tr>
<td>max-iteration</td>
<td>100</td>
<td>particle number</td>
<td>20</td>
</tr>
<tr>
<td>tolerance</td>
<td>$10^{-8}$</td>
<td>frequency locking</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>

$^aA_0, f_0$ and $\phi_0$ are initialised based on the solution of the pre-calculation.

**Table 5** Comparison of accuracies at the 60th generations

<table>
<thead>
<tr>
<th>MLI</th>
<th>AvRE $%$</th>
<th>$R\alpha E$ $%$</th>
<th>$R\gamma E$ $%$</th>
<th>$R\phi E$ $%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No MLI</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
</tr>
<tr>
<td>MLI</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
<td>1.68 x 10^{-5}</td>
</tr>
</tbody>
</table>

Fig. 6 Spectral information of the signal in this case
(a) Whole spectra, (b) Spectra around 50 Hz, (c) Spectra around 350 Hz

Fig. 7 Fitness of $p_{\text{best}}$ changes along with the growing iteration number
Considering the information in the frequency domain is too complex in this situation, the spectra of some generations are picked out for observation, which is shown in Fig. 9.

It can be observed that the difference between the blue spectra (reconstructed spectra $\mathbf{\bar{Q}_2}$) and the red spectra (practical spectra $\mathbf{Q}_2$) is large at the beginning. However, as the iteration number growing, the difference gradually gets less. In the numeric aspect, the AυRE versus the 10th generation are about $10^{-3}$ or $10^{-4}$, and the AυREs versus the 50th generation are about $10^{-4}$ or $10^{-5}$. The detailed values are listed in Table 6 and the whole iterating process is shown in Fig. 10.

Although the reducing speed of AυRE in Fig. 10 is lower than the one corresponding to Fig. 7, the accuracy at the tenth generation (RE $\approx 10^{-5}$) still satisfies the requirements of national and international standards.

4.3 Detection with a sharp amplitude-drop between two adjacent sinusoids

If the amplitude-disparity of two adjacent sinusoids is very large, the mainlobe of the amplitude-small one might be submerged in the mainlobe of the bigger one. In this situation, the proposed method is still workable. To verify this function, a signal is constructed, as (45). There are two sinusoids in this signal. The amplitude of the 45.7 Hz harmonic is over 20 times than the amplitude of the 55 Hz one

$$x(n) = 320 \sin(2\pi \times 45.7 \times n T_s + 87^\circ) + 11 \sin(2\pi \times 55 \times n T_s + 145^\circ)$$

After transforming via IpDFT with $T_0 R_{11}$ window, the precalculation solution can be obtained. The corresponding parameters are listed in Table 7, which also serve as the initial value in PSO, given in the equation below:

$$p_{\omega}^i(i) = [45.6816 \ 319.2721 \ 87.4985]$$

After optimisation, the final result is also listed in Table 7. The result is unexpected: REs after optimisation are generally worst than the ones before optimisation.

To find the reason, some spectra obtained via DFT, i.e. $\mathbf{Q}_2$, are shown in red in Fig. 11. Then, the spectra reconstructed based on the parameters in (46), i.e. $\mathbf{\bar{Q}_2}$, are shown in blue as the contrast. In Fig. 11, Fig. 11a displays amplitudes of the spectra and Fig. 11b displays their phases.

It can be observed: for the spectra reconstructed and the ones obtained via DFT, though the disparity on amplitudes is very little, their phases are different evidently. Hence, the reason to get a worst solution can be supposed: the parameters in (46) are only able to reconstruct one mainlobe. However, they are expected to reconstruct two according to the algorithm principle. This dilemma results in the final result not similar to either of the two sinusoids in (45). Hence, the accuracy goes worst.

<table>
<thead>
<tr>
<th>Env</th>
<th>Type</th>
<th>Sinusoids</th>
<th>RFE</th>
<th>Proposed</th>
<th>Relative amplitude error</th>
<th>Proposed</th>
<th>Relative phase error</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-noise MLI</td>
<td>Fun.</td>
<td>4.85 $\times 10^{-3}$</td>
<td>5.81 $\times 10^{-3}$</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>9.19 $\times 10^{-3}$</td>
<td>3.72 $\times 10^{-3}$</td>
<td>6.03 $\times 10^{-8}$</td>
<td>1.21 $\times 10^{-1}$</td>
</tr>
<tr>
<td>Inter-1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>---</td>
<td>---</td>
<td>3.28 $\times 10^{-6}$</td>
<td>---</td>
</tr>
<tr>
<td>Harm-7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>6.66 $\times 10^{-4}$</td>
<td>7.18 $\times 10^{-4}$</td>
<td>3.04 $\times 10^{-6}$</td>
<td>1.49 $\times 10^{-3}$</td>
</tr>
<tr>
<td>Harm-3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>5.97 $\times 10^{-3}$</td>
<td>2.36 $\times 10^{-2}$</td>
<td>5.78 $\times 10^{-7}$</td>
<td>1.52 $\times 10^{-2}$</td>
</tr>
<tr>
<td>Inter-4</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>1.06 $\times 10^{-3}$</td>
<td>1.95 $\times 10^{-5}$</td>
<td>4.78 $\times 10^{-11}$</td>
<td>3.83 $\times 10^{-13}$</td>
</tr>
<tr>
<td>Inter-1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>4.38 $\times 10^{-6}$</td>
<td>2.20 $\times 10^{-11}$</td>
<td>4.98 $\times 10^{-9}$</td>
<td>1.89 $\times 10^{-11}$</td>
</tr>
<tr>
<td>Harm-3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>4.38 $\times 10^{-6}$</td>
<td>7.68 $\times 10^{-14}$</td>
<td>3.35 $\times 10^{-5}$</td>
<td>1.27 $\times 10^{-9}$</td>
</tr>
<tr>
<td>Harm-9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>1.00 $\times 10^{-9}$</td>
<td>5.95 $\times 10^{-14}$</td>
<td>6.43 $\times 10^{-15}$</td>
<td>1.93 $\times 10^{-3}$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>5.81 $\times 10^{-3}$</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>9.19 $\times 10^{-3}$</td>
<td>3.72 $\times 10^{-3}$</td>
<td>6.10 $\times 10^{-9}$</td>
</tr>
<tr>
<td>Inter-1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>---</td>
<td>---</td>
<td>3.33 $\times 10^{-8}$</td>
<td>---</td>
</tr>
<tr>
<td>Harm-7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>6.67 $\times 10^{-4}$</td>
<td>7.19 $\times 10^{-4}$</td>
<td>1.88 $\times 10^{-7}$</td>
<td>1.49 $\times 10^{-3}$</td>
</tr>
<tr>
<td>Harm-3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>5.97 $\times 10^{-3}$</td>
<td>2.35 $\times 10^{-2}$</td>
<td>3.14 $\times 10^{-9}$</td>
<td>1.52 $\times 10^{-2}$</td>
</tr>
<tr>
<td>Inter-3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>9.32 $\times 10^{-8}$</td>
<td>1.70 $\times 10^{-5}$</td>
<td>2.07 $\times 10^{-6}$</td>
<td>8.36 $\times 10^{-6}$</td>
</tr>
<tr>
<td>Harm-9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>$&lt;1.00 \times 10^{-5}$</td>
<td>5.38 $\times 10^{-8}$</td>
<td>2.77 $\times 10^{-6}$</td>
<td>7.33 $\times 10^{-6}$</td>
<td>2.47 $\times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 5 REs of the final results compared with methods in papers [4–6]
Aiming at this question, the size of the initial particle is suggested to be \(2 \times 3\), and the new particle is given in (47). The iteration parameters of the first row refers to Table 4; meanwhile, the iteration parameters of the second row, i.e. the second sinusoid, see Table 8.

After re-iterating, the parameters and their REs are listed in Table 9. Compared to the result in Table 7, the new result contains better accuracy. Therefore, the size of the particle is suggested to be increased if the burden on time-consuming is not heavy.

5 Practical detection

The data acquired in some laboratory serves as the samples in this work. All modules are shown in Fig. 12. The current in phase-A was sampled via a pincer-shaped Rogowski coil [17] and portable engineering institutions. After sampling and synchronising, LabVIEW was used to receive and analyse this signal. Compared to using a demo board, this selection is more convenient to debug the programme of the proposed algorithm.

After applying the proposed method, the estimators of parameters versus each branch of a sinusoid are listed in Table 10. It can be noted that the amplitude of each harmonic is much less than the one of the fundamental wave, which is reasonable and consistent with the reality. However, the RE of each parameter cannot be calculated, because the authentic values of parameters are unavailable.


**Table 8** Configuration of iteration parameters for another sinusoid

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight $\omega_{SE}$</td>
<td>0.9</td>
<td>cell of $f$</td>
<td>$f_0 + 27^a$</td>
</tr>
<tr>
<td>weight $\omega_{ED}$</td>
<td>0.4</td>
<td>floor of $f$</td>
<td>$f_0 - 27^b$</td>
</tr>
<tr>
<td>constant $c_1$</td>
<td>1.2</td>
<td>cell of $A$</td>
<td>$A_0$</td>
</tr>
<tr>
<td>constant $c_2$</td>
<td>1.3</td>
<td>floor of $A$</td>
<td>0</td>
</tr>
<tr>
<td>max-iteration</td>
<td>100</td>
<td>cell of $\varphi$</td>
<td>$180^*$</td>
</tr>
<tr>
<td>particle number</td>
<td>20</td>
<td>floor of $\varphi$</td>
<td>$-180^*$</td>
</tr>
<tr>
<td>tolerance</td>
<td>$10^{-8}$</td>
<td>frequency locking</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>

*Twenty-seven is the half-mainlobe width in this case.

**Table 9** Solutions after optimisation based on size-different particles

<table>
<thead>
<tr>
<th>Result</th>
<th>Fre., Hz</th>
<th>Amp., mA</th>
<th>Pha., deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>With 1×3 initial matrix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wave-1 Value</td>
<td>45.6900</td>
<td>318.5928</td>
<td>85.1393</td>
</tr>
<tr>
<td>RE</td>
<td>$2.19 \times 10^{-4}$</td>
<td>$4.40 \times 10^{-3}$</td>
<td>$2.14 \times 10^{-2}$</td>
</tr>
<tr>
<td>With 2×3 initial matrix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wave-1 Value</td>
<td>45.7000</td>
<td>319.9501</td>
<td>86.9864</td>
</tr>
<tr>
<td>RE</td>
<td>$&lt;1 \times 10^{-5}$</td>
<td>$1.56 \times 10^{-4}$</td>
<td>$1.57 \times 10^{-4}$</td>
</tr>
<tr>
<td>Wave-2 Value</td>
<td>55.0000</td>
<td>10.9998</td>
<td>145.0001</td>
</tr>
<tr>
<td>RE</td>
<td>$&lt;1 \times 10^{-5}$</td>
<td>$1.52 \times 10^{-5}$</td>
<td>$7.87 \times 10^{-7}$</td>
</tr>
</tbody>
</table>


After detection, each branch of harmonic can be filtered. Then, a new signal can be reconstructed based on the data in Table 10. The details on the wave peak in the time domain are shown in Fig. 13. Compared to the original signal, the reconstructed signal contains fewer irregular components, thus it becomes smoother.

Spectra in the frequency domain are analysed as well. The frequency-domain information obtained via T₃R₁₁ window is shown in Fig. 14a, and the information versus a common IpDFT is shown in Fig. 14b. It can be observed that the T₃R₁₁ window contains a wider mainlobe width. In the meantime, the sidelobes of this window stay at a lower-value degree. This phenomenon is helpful to restrain the interference from the fundamental wave sidelobes to harmonics. By using this window, the mainlobes of harmonics are more likely to appear distinctly. Therefore, it is feasible to apply T₃R₁₁ window to acquire a high detection accuracy.

6 Conclusion

A novel method of inter-harmonics detection and parameters estimation is proposed, which contains two steps: in the pre-calculation, an IpDFT-based method with T₃R₁₁ window is applied to estimate an approximate solution. Then, this solution is optimised via the accelerated PSO.

For the pre-calculation: a specific T₃R₁₁ window is applied. It contains very low sidelobes, whose loudness degree is about $-226\, {\text{dB}}$. This value could restrain the error caused by spectral leakage to a degree of $10^{-11}$. Meanwhile, the mainlobe of this window is a little wide, which makes it possible to choose the outside spectra to construct the interpolation formula. This selection is necessary because outside spectra are impacted more slightly than the ones around the peak when MLI exists.

For the accelerated PSO: three aspects of a traditional PSO have been improved to accelerate the algorithm. First, both the Rect. window and the Tria. window only produce one vector at a point in the frequency domain. This character is utilised to reduce the calculation amount in the equations from (28) to (31). Second, the frequency becomes locked when the RfE becomes $<10^{-5}$. This strategy could reduce the particle dimensions by a factor of 1/3. Finally, a changing inertia weight is applied, which is helpful to avoid the local-optimum solution and to rise the speed to approach the best solution.

In the aspect of detection accuracy: the AvRE of the parameters is limited to a degree of $1.68 \times 10^{-3}$ at the tenth generation, and it can be even optimised to a degree of $10^{-7}$ with rising the iteration number. In general, both the accuracy and the calculating speed of the method satisfy the requirement of a power grid. In the coming research, algorithms about de-noising [22] or about rising the anti-noise capability are both considerable to rise the accuracy further.

7 Acknowledgments

This work was supported by:

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ii. The National Natural Science Foundation of China. The Grant no. is 51507091.
Table 10 Parameters detected in the practical application

<table>
<thead>
<tr>
<th>Order</th>
<th>Freq., Hz</th>
<th>Amp., mA</th>
<th>Pha., deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.040</td>
<td>191.513</td>
<td>152.727</td>
</tr>
<tr>
<td>2</td>
<td>103.579</td>
<td>0.074</td>
<td>286.288</td>
</tr>
<tr>
<td>3</td>
<td>152.004</td>
<td>0.692</td>
<td>82.032</td>
</tr>
<tr>
<td>4</td>
<td>201.188</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>5</td>
<td>250.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>6</td>
<td>300.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>7</td>
<td>350.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>8</td>
<td>400.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>9</td>
<td>450.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>10</td>
<td>500.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>11</td>
<td>550.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>12</td>
<td>600.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>13</td>
<td>650.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
<td>14</td>
<td>700.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
<tr>
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<td>0.108</td>
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<tr>
<td>17</td>
<td>850.344</td>
<td>0.108</td>
<td>235.874</td>
</tr>
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</table>

Fig. 12 Online detection instruments. In this sampling system, the sampling frequency is 4000 Hz. About 1000 points were sampled from each phase. The data in phase-A was employed as the signal to be detected in this work: (a) Spectra from T5R11 window, (b) Spectra from common IpDFT

Fig. 13 Detailed comparison between the original signal and the reconstruction signal

Fig. 14 Capability of T3R1 window in restraining the interference from sidelobes: (a) Spectra obtained using the proposed method, (b) Ones transformed via a common IpDFT

8 References

9 Appendix

The expression of an odd-length Tria. window in the time domain is given in (48)

\[ w_T(m) = 1 - \frac{2m - (M + 1)}{M + 1} \]  

(48)

Moreover, the corresponding DFT is given in (49)

\[ W_T(k) = \frac{2 \sin[k\pi(M + 1)/2M] e^{-j\pi M}}{(M + 1) \sin(k\pi/M)} \]  

(49)

All variables in these two equations are the same with (2) and (3). Actually, the difference between (5) and (49) is very slight, when the length \( M \) is very large. However, errors caused by this difference is still noteworthy that since the accuracy requirement for harmonic detection is extremely high.