Distributed Resource Allocation for Proportional Fairness in Multi-Band Wireless Systems

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***Abstract*—A challenging problem in multi-band multi-cell self-organized wireless systems, such as multi-channel Wi-Fi networks, femto/pico cells in 3G/4G cellular networks, and more recent wireless networks over TV white spaces, is of distributed resource allocation. This involves four components: channel selection, client association, channel access, and client scheduling. In this paper, we present a unified framework for jointly addressing the four components with the global system objective of maximizing the clients throughput in a proportionally fair manner. Our formulation allows a natural dissociation of the problem into two sub-parts. We show that the first part, involving channel access and client scheduling, is convex and derive a distributed adaptation procedure for achieving Pareto-optimal solution. For the second part, involving channel selection and client association, we develop a Gibbs-sampler based approach for local adaptation to achieve the global objective, as well as derive fast greedy algorithms from it that achieve good solutions.**

I. Introduction

Many of the existing and evolving wireless systems operate over spectrum that spans multiple bands. These bands may be contiguous, as in OFDM-based systems, such as current IEEE 802.11-based WLANs (a.k.a. Wi-Fi networks) and evolving fourth-generation LTE cellular wireless systems; or they may be spread far apart, as in multi-channel 802.11 systems and in recently proposed wireless broadband networks over TV white spaces (discussed in the sequel). A common issue in these multi-band systems is how to perform resource allocation among different clients, possibly being served by different access points (APs). This needs to be done so as to efficiently utilize wireless resources—spectrum, transmission opportunities and power—while being fair to different clients. Furthermore, unlike traditional enterprise Wi-Fi networks and cellular wireless networks, where the placement of APs and their operating bands are arrived at after careful capacity/coverage planning, more and more of the evolving wireless systems are going to be self-organized networks. There is extensive literature on completely self-organized wireless networks, also referred to as *ad hoc* networks [10], which are often based on 802.11. Even the emerging 4G cellular wireless networks, such as those based on LTE, are going to have a significant deployment of self-organized subsystems: namely, pico cells and femto cells [2]. These self-organized (sub)-systems will require that resource allocation is performed dynamically in a distributed manner and with minimal coordination between different APs and/or clients.

In this paper, we consider the problem of joint resource allocation across different APs and their clients so as to achieve a global objective of maximizing the system throughput while being fair to users. We focus on achieving proportional fairness, which has become essentially standard across current 3G cellular systems, as well as in emerging 4G systems based on LTE and WiMAX. Thus, the system objective will be to allocate wireless resources, spectrum and transmission opportunities, so as to maximize the (weighted) sum of the log of throughputs of different clients, which is known to achieve (weighted) proportional fairness.

The combined resource allocation in a multi-band multi-cell wireless system involves four components: Channel Selection, Client Association, Channel Access, and Client Scheduling. The first component, Channel Selection, decides on how different APs share different bands of the spectrum available to the wireless system. The second component, Client Association, allows a client to decide on an AP to associate within its neighborhood that is likely to provide the "best" performance. Once an AP has chosen a channel/band to operate in and a bunch of clients have associated with it, the third component, Channel Access, decides when it should access the channel so as to serve its clients while being fair to other access points in its neighborhood operating in the same channel. The final component, Client Scheduling, decides which of its clients an AP should serve whenever it successfully accesses the channel.

Our approach addresses the four components in a unified framework, where the solutions to different components are arrived at through separation of time scales of adaptation. More specifically, our formulation allows for the optimization problem of maximizing the clients throughput with weighted proportional fairness to naturally dissociate into two sub-parts, which are adapted at different time scales. Assuming that the channel selection and client association have been performed, we show that the sub-problem of channel access and client scheduling becomes convex, which is also amenable to a distributed adaptation for achieving Pareto-optimal weighed proportional fairness. At a slower time scale, we adapt the channel selection and client association to varying demands and interference. This part is a non-convex problem in general, and thus, difficult to solve for a globally optimal solution. We develop a Gibbs-sampler based approach to perform local adaptation while improving the global system objective. The adaptation is randomized, and if done slowly enough, can achieve a globally optimal solution. In practice, however, that may not be always feasible; hence, we derive greedy heuristics from it for channel selection and client association, which though not globally optimal, provide fast and good distributed solutions with limited exchange of information, as the simulation results indicate. Simulation results also indicate that our policies achieve better performance than state-of-the-art techniques, such as those proposed in [8] and [9].

The paper is organized as follows. Section II describes the multi-cell multi-band wireless system model and the joint resource allocation problem that we study. Section III gives an overview of our approach and discusses the separation of problems. The details of our approach and its desirable properties, including convergence to Pareto-optimal proportionally-fair allocation, are given in Section IV and Section V. The results of simulations are provided in Section VI. Concluding remarks are discussed in Section VII.

II. System Model

We consider a system with several access points (APs) and clients that can operate in a number of channels. We denote the set of APs by $\mathbb{N}$, the set of clients by $\mathbb{I}$, and the set of channels by $\mathbb{C}$. Each client i is associated with an AP, which we denote by $n(i)$, and is served by that AP. Each AP n is equipped with u_n radios that can operate in different channels simultaneously and each client i is equipped with only one radio. When an AP n has more than one radios, i.e, $u_n > 1$, we can simplify the model by assuming that there are u_n APs, each with one radio, that are placed at the same place as AP n. Each of these u_n APs corresponds to one radio of AP n. This procedure allows us to only consider the model in which each AP has one radio and simplifies notations. Throughout the rest of the paper, we assume that each AP only has one radio and operates in one channel unless otherwise specified. The channel that an AP n is operating in is denoted by $c(n)$. APs can switch the channels that they operate in, although such switches can only be done infrequently due to the large overheads. When an AP switches channels, all its clients also switch channels accordingly.

We focus on a server-centric scheme where each AP schedules all transmissions between itself and all clients that are associated with it. This scheme is applicable to a wide varieties of wireless systems that include LTE, WiMax, and IEEE 802.11 PCF[1]. We assume that time is slotted, with the duration of a time slot equals the time needed for a transmission. If an AP n makes a successful transmission toward client i in channel c, client i receives data at rate of $B_{i,n,c}$ in this slot. Since characteristics of different channels may be different, $B_{i,n,c}$ depends on c.

On the other hand, we assume that the APs are not synchronized and may interfere with each other. We consider the interference relations using the protocol model [5]. When an AP n operates in channel c, it may be interfered by a subset $\mathcal{M}^{n,c}$ of APs, where $n \in \mathcal{M}^{n,c}$ for notational simplicity. When the AP n schedules a transmission between itself and one of its clients, the transmission is successful if n is the only AP among $\mathcal{M}^{n,c}$ that transmits in channel c during the time of transmission; otherwise, the transmission suffers from a collision and fails. We assume that the interference relations are symmetric, i.e., $m \in \mathcal{M}^{n,c}$ if and only if $n \in \mathcal{M}^{m,c}$. Note that, since the propagation characteristics of different channels may be different, especially in the case of TV white space access, the subset $\mathcal{M}^{n,c}$ depends on the channel c. This dependency further distinguishes our work from most existing works on multi-channel access where interference relations are assumed to be identical for all channels. We also define $\mathcal{N}_i$ to be the set of APs that client i can be associated with, and $\mathcal{M}_i := \{m \in \mathbb{N} | \exists c \in \mathbb{C}, n \in \mathcal{N}_i \text{ s.t. } m \in \mathcal{M}^{n,c}\}$ for each client i.

Since APs are not coordinated, we assume that they access the channel by random access. Each AP n chooses a random access probability, p_n. In each slot, AP n accesses the channel $c(n)$ with probability p_n. The transmission is successful if n is the only AP in $\mathcal{M}^{n,c(n)}$ that transmits over channel $c(n)$. Thus, the probability that AP n successfully accesses the channel can be expressed as

$$p_n \prod_{\substack{m\in\mathcal{M}^{n,c(n)},\\ m\neq n, c(m)=c(n)}} (1-p_m) = \frac{p_n}{1-p_n} \prod_{\substack{m\in\mathcal{M}^{n,c(n)},\\ c(m)=c(n)}} (1-p_m). \quad (1)$$

The AP is in charge of scheduling transmissions for its clients. When AP n accesses the channel, it schedules the transmission for client i, where $n(i) = n$, with probability $\phi_{i,n}$, $\phi_{i,n} \geq 0$ and $\sum_{i:n(i)=n} \phi_{i,n} = 1$. Since the data rate of client i when it is served is $B_{i,n(i),c(n(i))}$ and the probability that the AP $n(i)$ makes a successful transmission is as in Eq. (1), its throughput per time slot is, assuming $n(i) = n$ and $c(n) = c$,

$$r_i := B_{i,n,c}\phi_{i,n} \frac{p_n}{1-p_n} \prod_{m\in\mathcal{M}^{n,c}, c(m)=c} (1-p_m). \quad (2)$$

Finally, we assume that each client is associated with a positive weight $w_i > 0$. We also denote the total weights of clients associated with AP n by w^n, i.e., $w^n := \sum_{i:n(i)=n} w_i$. The goal is to achieve weighted proportional fairness among clients, that is, to maximize $\sum_{i\in\mathbb{I}} w_i \log r_i$.

III. Solution Overview and Time-Scale Separation

We now give an overview of our approach to achieve weighted proportional fairness, which consists of separating the problem into four components and solving them. By Eq. (2), we can formulate the problem of achieving weighted proportional fairness as the following optimization problem:

$$\begin{aligned}
\text{Max } & \textstyle\sum_i w_i \log r_i \\
&= \textstyle\sum_i w_i [\log B_{i,n(i),c(n(i))} + \log \phi_{i,n(i)} + \log \frac{p_{n(i)}}{1-p_{n(i)}} \\
&\quad + \log \textstyle\prod_{m\in\mathcal{M}^{n(i),c(n(i))}, c(m)=c(n(i))} (1-p_m)], \\
\text{s.t. } & c(n) \in \mathbb{C}, \text{ for all } n; \quad n(i) \in \mathcal{N}_i, \text{ for all } i; \\
& 0 \leq p_n \leq 1, \text{ for all } n; \quad \phi_{i,n(i)} \geq 0, \text{ for all } i; \\
& \textstyle\sum_{i:n(i)=n} \phi_{i,n} = 1, \text{ for all } n.
\end{aligned}$$

Based on this formulation, the problem of achieving weighted proportional fairness consists of four important components, in increasing order of time scales: First, whenever the AP accesses the channel, it needs to schedule one client for service. That is, the AP has to decide the values of $\phi_{i,n}$. Second, in each time slot, the AP has to decide whether it should access the channel, which consists of determining the values of p_n. Third, each client needs to decide which AP it should be associated with, i.e., deciding $n(i)$. Finally, each AP n needs to choose a channel, $c(n)$, to operate in. We denote the four components as *Scheduling Problem*, *Channel Access Problem*, *Client Association Problem*, and *Channel Selection*

[1]A distributed scheme, which is applicable to IEEE 802.11 DCF, is discussed in the technical report [7].

Problem, respectively. Weighted proportional fairness is achieved by jointly solving the four components.

Since the overhead for a client to change the AP it is associated with and for an AP to change the channel it operates in are high, solutions to the Client Association Problem and the Channel Selection Problem are updated at a much slower time scale compared to solutions to the Scheduling Problem and the Channel Access Problem. Based on this timescale separation, we first study the solutions to the Scheduling Problem and the Channel Access Problem, given fixed solutions to the Client Association Problem and the Channel Selection Problem. We then study the solutions to the Client Association Problem and the Channel Selection Problem, under the knowledge of how solutions to the Scheduling Problem and the Channel Access Problem react. Thus, solutions to the Client Association Problem and the Channel Selection Problem are indeed joint solutions to all the four components, and their optimal solutions achieve Pareto-optimal weighted proportional fairness. In addition to solving the four components, we will show that the solutions naturally turn into distributed algorithms where each client/AP makes decisions based on local knowledge.

IV. The Scheduling Problem and the Channel Access Problem

In this section, we assume that solutions to the Client Association Problem and the Channel Selection Problem, i.e., $n(i)$ and $c(n)$, are fixed.

Since $n(i)$ and $c(n)$ are fixed, values of $B_{i,n(i),c(n(i))}$ are constant. The optimization problem can be rewritten as

$$\begin{aligned}\text{Max } & \textstyle\sum_{i\in\mathbb{I}} w_i[\log \phi_{i,n(i)} + \log \frac{p_{n(i)}}{1-p_{n(i)}} \\ & \quad + \log \prod_{m\in\mathcal{M}^{n(i),c(n(i))},c(m)=c(n(i))}(1-p_m)] \\ & = \textstyle\sum_{i\in\mathbb{I}} w_i \log \phi_{i,n(i)} + \sum_{n\in\mathbb{N}}[w^n \log p_n \\ & \quad + (\textstyle\sum_{m\in\mathcal{M}^{n,c(n)},c(m)=c(n)} w^m - w^n)\log(1-p_n)], \\ \text{s.t. } & 0 \le p_n \le 1, \text{ for all } n; \quad \phi_{i,n(i)} \ge 0, \text{ for all } i; \\ & \textstyle\sum_{i:n(i)=n} \phi_{i,n} = 1, \text{ for all } n,\end{aligned}$$

where $w^n = \sum_{i:n(i)=n} w_i$, as defined in Section II. We also define $z^n := \sum_{m\in\mathcal{M}^{n,c(n)},c(m)=c(n)} w^m$ to be the total weights of clients that are associated with APs that interfere with n, including itself.

This formulation naturally decomposes the optimization problem into two independent parts: maximizing $\sum_i w_i \log \phi_{i,n(i)}$ over $\phi_{i,n}$, which is the Scheduling Problem, and maximizing $\sum_n [w^n \log p_n + (z^n - w^n)\log(1-p_n)]$ over p_n, which is the Channel Access Problem. Thus, we can solve these two problems independently.

We first solve the Scheduling Problem by the following theorem, whose proof is included in [7].

Theorem 1: Given $n(i)$, $c(n)$, and p_n, for all i and n, $\sum_i w_i \log r_i$ is maximized by setting $\phi_{i,n(i)} \equiv w_i/w^{n(i)}$.

We solve the Channel Access Problem next. The following theorem is the direct result of Theorem 1 in [6].

Theorem 2: Given $n(i)$, $c(n)$, and $\phi_{i,n}$, for all i and n, $\sum_i w_i \log r_i$ is maximized by setting $p_n \equiv w^n/z^n$.

In summary, when the solutions to the Client Association Problem and the Channel Selection Problem, i.e., $n(i)$ and $c(n)$, are fixed, the AP n should access the channel with probability $p_n = w^n/z^n$ in each time slot and should schedule the transmission for its client i with probability $\phi_{i,n} = w_i/w^n$ whenever it accesses the channel. In addition to achieving the optimal solution to both the Scheduling Problem and the Channel Access Problem, this solution only requires n to know the local information of w^m and $c(m)$ for all AP m that may interfere with itself. Thus, this solution can be easily implemented in a distributed manner.

V. The Client Association Problem and the Channel Selection Problem

We now propose a distributed algorithm that solves the Client Association Problem and the Channel Selection Problem based on the knowledge of optimal solutions to the Scheduling Problem and the Channel Access Problem. These two problems are non-convex and a local optimal solution to the two problems may not be globally optimum, which we will also illustrate by simulations in Section VI. Thus, common techniques for solving convex problems are not suitable for these problems. Instead, the proposed algorithm uses a simulated annealing technique that is based on the Gibbs Sampler [4], which is proven to converge to the global optimum point almost surely. In the sequel, we also derive a greedy heuristic that is easier to implement and converges faster.

We call a joint solution to both the Client Association Problem and the Channel Selection Problem as a *configuration* of the system. A configuration is thus fully specified by the AP each client is associated with, and the channel each AP operates in. Define ψ_t as the configuration of the system at time t. We define the *utility* of the system under configuration ψ_t, which we denote by $U(\psi_t)$, as the value of $\sum_i w_i \log r_i$ when APs and clients choose their channels to operate in and APs to be associated with according to ψ_t, and apply the optimal solution to the Scheduling Problem and the Channel Access Problem under ψ_t. We then have

$$\begin{aligned}U(\psi_t) = \; & \textstyle\sum_{i\in\mathbb{I}} w_i[\log B_{i,n(i),c(n(i))} + \log \frac{w_i}{w^{n(i)}}] \\ & + \textstyle\sum_{n\in\mathbb{N}}[w^n \log \frac{w^n}{z^n} + (z^n - w^n)\log \frac{z^n - w^n}{z^n}].\end{aligned} \tag{3}$$

Finding the joint solution that achieves Pareto-optimal proportional fairness is equivalent to finding the configuration ψ that maximizes $U(\psi)$.

We apply the Gibbs sampler to solve the Client Association Problem and the Channel Selection Problem jointly. At each time t, either a client or an AP is selected in a random or round-robin fashion. The selected client, or AP, then changes the AP it is associated with, or the channel it operates in, randomly, while all other clients and APs make no changes. The solutions to the Scheduling Problem and the Channel Access Problem are then updated according to the new configuration.

We now discuss how the selected client, or AP, changes the AP it is associated with, or the channel it operates in. Let $\psi_t(n(i) = n)$ be the configuration where client i is associated with AP n, and the remaining of the system is the same as in configuration ψ_t. We can define $\psi_t(c(n) = c)$ for AP n similarly. If client i is selected at time t, it changes the AP it is associated with to n with probability $e^{U(\psi_t(n(i)=n))/T(t)} / \sum_m e^{U(\psi_t(n(i)=m))/T(t)}$, where $T(t)$ is a positive decreasing function. On the other hand, if AP n is selected at time t, it changes the channel it operates in to c with probability $e^{U(\psi_t(c(n)=c))/T(t)} / \sum_d e^{U(\psi_t(c(n)=d))/T(t)}$.

It remains to compute the values of $U(\psi_t(n(i) = n))$ for client i and $U(\psi_t(c(n) = c))$ for AP n. We first discuss how to compute $U(\psi_t(n(i) = n))$. Let $w^n_{-i} := \sum_{j:n(j)=n,j\ne i} w_j$

be the total weights of clients, excluding i, associated with AP n. Let $z^n_{-i} := \sum_{m\in\mathcal{M}^{n,c(n)},c(m)=c(n)} w^m_{-i}$. Define

$$U^0_i(\psi_t) = \sum_{j\in\mathbb{I},j\neq i} w_j[\log B_{j,n(j),c(n(j))} + \log \frac{w_j}{w^{n(j)}_{-i}}] + \sum_{n\in\mathbb{N}}[w^n_{-i}\log\frac{w^n_{-i}}{z^n_{-i}} + (z^n_{-i} - w^n_{-i})\log\frac{z^n_{-i}-w^n_{-i}}{z^n_{-i}}],$$

which can be thought of as the utility of the system as if the weight of client i were zero. We then define $\Delta U^n_i(\psi_t) := U(\psi_t(n(i) = n)) - U^0_i$. Since in the configuration $\psi_t(n(i) = n)$, $w^m = w^m_{-i}$ for all $m \neq n$; $w^n = w^n_{-i} + w_i$; $z^m = z^m_{-i} + w_i$ if $m \in \mathcal{M}^{n,c(n)}$, $c(m) = c(n)$, and $m \neq n$; and $z^m = z^m_{-i}$, otherwise, we have

$$\begin{aligned}
&\Delta U^n_i(\psi_t)\\
&= w_i[\log B_{i,n,c(n)} + \log\frac{w_i}{w^n}] + \sum_{j:n(j)=n} w_j \log\frac{w^n_{-i}}{w^n_{-i}+w_i}\\
&+ w_i\log\frac{w^n}{z^n} + w^n_{-i}\log\frac{z^n_{-i}(w^n_{-i}+w_i)}{w^n_{-i}(z^n_{-i}+w_i)} + (z^n_{-i} - w^n_{-i})\log\frac{z^n_{-i}}{z^n_{-i}+w_i}\\
&+ \sum_{m\in\mathcal{M}^{n,c(n)},m\neq n,c(m)=c(n)}[w^m_{-i}\log\frac{z^m_{-i}}{z^m_{-i}+w_i}\\
&(z^m_{-i} - w^m_{-i})\log\frac{(z^m_{-i}-w^m_{-i}+w_i)z^m_{-i}}{(z^m_{-i}+w_i)(z^m_{-i}-w^m_{-i})} + w_i\log\frac{z^m-w^m}{z^m}]\\
&= w_i[\log\frac{B_{i,n,c(n)}w_i}{z^n} + \sum_{\substack{m\in\mathcal{M}^{n,c(n)},\\ m\neq n,c(m)=c(n)}}\log\frac{z^m-w^m}{z^m}]\\
&+ \sum_{\substack{m\in\mathcal{M}^{n,c(n)},\\ m\neq n,c(m)=c(n)}}\log[(1+\frac{w_i}{z^m_{-i}-w^m_{-i}})^{z^m_{-i}-w^m_{-i}}/(1+\frac{w_i}{z^m_{-i}})^{z^m_{-i}}]\\
&+ \log(1-\frac{w_i}{z^n_{-i}+w_i})^{z^n_{-i}}\\
&\approx w_i[\log(\frac{B_{i,n,c(n)}w_i}{z^n}\prod_{\substack{m\in\mathcal{M}^{n,c(n)},\\ m\neq n,c(m)=c(n)}}\frac{z^m-w^m}{z^m})] + \alpha,
\end{aligned}$$

where α is a constant. Since $(1 + \frac{w_i}{A})^A \approx e^{w_i}$ and $(1 - \frac{w_i}{A})^A \approx e^{-w_i}$ for all $A >> w_i$, the last approximation holds when $z^m_{-i} >> w_i$, which is true in a dense network where the weights of all clients are within the same order.

Suppose a client i is selected to change its state at time t, at which time the configuration of the system is ψ_t. The probability that i chooses AP n to be associated with is $e^{[U^0_i(\psi_t)+\Delta U^n_i(\psi_t)]/T(t)}/\sum_m e^{[U^0_i(\psi_t)+\Delta U^m_i(\psi_t)]/T(t)} \approx (\frac{B_{i,n,c(n)}w_i}{z^n}\prod_{\substack{m\in\mathcal{M}^{n,c(n)},\\ m\neq n,c(m)=c(n)}}\frac{z^m-w^m}{z^m})^{w_i/T(t)}/\gamma$, where γ is the normalizer.

To compute the probability of choosing AP n to be associated with, client i only needs the values of $B_{i,n,c(n)}$ for all $n \in \mathcal{N}_i$, w^m and z^m for all $m \in \mathcal{M}_i$. Thus, this probability can be computed by client i using its local information. We also note that this probability has the following properties: First, it increases with $B_{i,n,c(n)}$, meaning that client i tends to choose the AP that has higher data rate; Second, it decreases with z^n, which is the total weights of clients that interfere with n; Finally, it increases with $\prod_{m\in\mathcal{M}^{n,c(n)},m\neq n,c(m)=c(n)}\frac{z^m-w^m}{z^m}$, which is the probability that none of the APs that interfere with n access the channel in a time slot. Thus, this probability jointly considers the three important factors for the Client Association Problem: data rate, interference, and channel congestion.

Next we discuss the computation of the probability that an AP n should choose channel c to operate in, if it is selected. Let $z^m_{-n} := \sum_{o\in\mathcal{M}^{m,c(m)},c(o)=c(m),o\neq n} w^o - w^m$. Let $U^0_n(\psi_t)$ be the utility of the system under configuration ψ_t, if the weights of all its clients were zero. That is,

$$U^0_n(\psi_t) = \sum_{j\in\mathbb{I},n(j)\neq n} w_j[\log B_{j,n(j),c(n(j))} + \log\frac{w_j}{w^{n(j)}}] + \sum_{m\in\mathbb{N},m\neq n}[w^m\log\frac{w^m}{z^m_{-n}} + (z^m_{-n} - w^m)\log\frac{z^m_{-n}-w^m}{z^m_{-n}}].$$

We then define $\Delta U^c_n(\psi_t) := U(\psi_t(c(n) = c)) - U^0_n$. Since in the configuration $\psi_t(c(n) = c)$, $z^m = z^m_{-n} + w^n$ if $m \in \mathcal{M}^{n,c(n)}$, $c(m) = c$, and $m \neq n$; and $z^m = z^m_{-n}$, otherwise, we have

$$\begin{aligned}
&\Delta U^c_n(\psi_t)\\
&= \sum_{i:n(i)=n} w_i[\log B_{i,n,c} + \log\frac{w_i}{w^n}]\\
&+ w^n\log\frac{w^n}{z^n} + (z^n - w^n)\log\frac{z^n-w^n}{z^n}\\
&+ \sum_{m\in\mathcal{M}^{n,c(n)},m\neq n,c(m)=c(n)}[w^m\log\frac{z^m_{-n}}{z^m_{-n}+w^n}\\
&(z^m_{-n} - w^m)\log\frac{(z^m_{-n}-w^m+w^n)z^m_{-n}}{(z^m_{-n}+w^n)(z^m_{-n}-w^m)} + w^n\log\frac{z^m_{-n}-w^m+w^n}{z^m_{-n}+w^n}].
\end{aligned}$$

When an AP n is selected by the Gibbs sampler at time t, it changes the channel it operates randomly, with the probability of changing to channel c proportional to $e^{U(\psi_t(c(n)=c))/T(t)} = e^{[U^0_n(\psi_t)+\Delta U^c_n(\psi_t)]/T(t)} \propto e^{\Delta U^c_n(\psi_t)/T(t)}$. We note that, to compute $\Delta U^c_n(\psi_t)$, AP n only needs the values of $B_{i,n,c}$, w_i, for each client i that is associated with n, and z^m_{-n}, w^m for all $m \in \cup_c\mathcal{M}^{n,c}$. Thus, $\Delta U^c_n(\psi_t)$ can also be computed using only local information.

Based on the above discussion, it is straightforward to design a distributed protocol (DP) using the Gibbs sampler. In DP, all clients and APs in the system only need to exchange information within their local areas, as they only need local information to compute the probability of choosing an AP to be associated with or a channel to operate in. Thus, DP is easily scalable. Further, DP achieves the Pareto-optimal proportional fairness as $t \to \infty$ almost surely by the following theorem derived from [4].

Theorem 3: If $T(t)$ satisfies the following conditions:

1) $T(t) \to 0$, as $t \to \infty$;
2) $T(t)\log t \to \infty$, as $t \to \infty$;

then $\lim_{t\to\infty} U(\psi_t) = \max_\psi U(\psi)$ with probability 1, for any initial configuration ψ_1.

In addition to DP, we can also consider a greedy policy (Greedy) that is easier to implement and converges faster. Greedy works similar to DP, except that when a client i, or an AP n, is selected by the sampler, it chooses the AP that maximizes $U(\psi_t(n(i) = n))$ to be associated with, or the channel that maximizes $U(\psi_t(c(n) = c))$ to operate in, respectively. It is essentially a steepest descent direction approach and is guaranteed to converge to a local optimal point. In addition to simple implementation, Greedy is also consistent with the selfish behavior of clients. Each client i chooses the AP n that maximizes $\frac{B_{i,n,c(n)}}{z^n}\prod_{o\in\mathcal{M}^{n,c(n)},c(o)=c(n),o\neq n}\frac{z^o}{z^o+w^o}$, which is indeed the value of r_i when i is associated with n. Thus, in Greedy, every client always chooses to associate with the AP that offers the highest throughput. The pseudo-codes of DP and Greedy are included in the technical report [7].

VI. Simulation Results

We have implemented both DP and Greedy algorithms and compared them against policies that use state-of-the-

id	frequency	bandwidth	id	frequency	bandwith
A	524 MHz	12 MHz	E	659 MHz	6 MHz
B	593 MHz	6 MHz	F	671 MHz	6 MHz
C	608 MHz	12 MHz	G	683 MHz	6 MHz
D	641 MHz	6 MHz			

TABLE I: List of white spaces in New York City.

art techniques for solving the Client Association Problem and the Channel Selection Problem. We compare with [8], which proposes a distributed algorithm for achieving minimum total interference among APs, for the Channel Selection Problem. For the Client Association Problem and the Scheduling Problem, we compare with two techniques. The first technique uses a Wifi-like approach where clients are associated with the closest AP and the AP schedules clients so that the throughput of each client is the same. The protocol that applies both [8] and the Wifi-like approach is called *MinInt-Wifi*. The other technique is one that is proposed in [9], which, under a fixed solution of the Channel Selection Problem, is a centralized algorithm that aims to find the joint optimal solution to the Client Association Problem and the Scheduling Problem that achieves weighted proportional fairness. This technique first relaxes the Client Association Problem by assuming that each client can be associated with more than one APs and formulates the problem as a convex programming problem. It then rounds up the solution to the convex programming problem and finds a solution to the Client Association Problem where each client is associated with only one AP. For ease of comparison, we use the solutions to the relaxed convex programming problem, which is indeed an upper-bound on the performance of [9]. The protocol that applies both [8] and [9] is called *MinInt-PF*. The Channel Access Problem is then solved by the optimal solutions based on the resulting solutions of MinInt-Wifi and MinInt-PF, respectively.

We compare the policies on two metrics: the weighted sum of the logarithms of throughput for clients, $\sum_{i\in\mathbb{I}} w_i \log r_i$, and the total weighted throughput $\sum_{i\in\mathbb{I}} w_i r_i$. All reported data are the average over 20 runs.

We consider a system consisting of 16 APs that are placed on a 4 by 4 grid. Each AP has 2 radios and adjacent APs are separated by 300 meters. There are 16 clients uniformly distributed in each of the two sectors $[0, 300] \times [0, 300]$ and $[600, 900] \times [600, 900]$; There are 9 clients uniformly distributed in each of the two sectors $[0, 300] \times [600, 900]$ and $[600, 900] \times [0, 300]$. We consider the TV white spaces available in New York City [3]. The list of available channels is shown in Table I. The data rates between APs and clients, and interference relations among APs, for each channel are derived from the ITU path loss model [1] and the simulation settings in [9]. We consider two settings: an unweighted setting where all clients have weights 1.0, and a weighted setting where clients within the region $[0, 300]\times[0, 900]$ have weights 1.5 and clients outside this region have weights 0.5.

Simulation results are shown in Fig. 1. For both the unweighted and weighted settings, MinInt-Wifi and MinInt-PF are far from optimum. The total weighted throughputs achieved by the two policies are less than half of those achieved by DP under both settings. These results show that policies that optimize the four components independently can result in poor performance. The performance

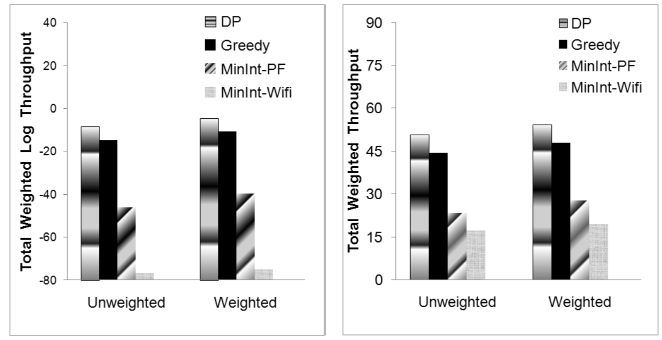

Fig. 1: Performance comparison.

of Greedy is close to optimum, whose weighted total throughputs are about 88% of those by DP for both the unweighted and weighted settings.

VII. Conclusion

We have studied the problem of achieving weighted proportional fairness in multi-band wireless networks. We have considered a system that consists of several APs and clients operating in a number of available channels, accounting for interference among APs and heterogeneous characteristics of different channels. We have identified that the problem of achieving weighted proportional fairness in such a system involves four important components: client scheduling, channel access, client association, and channel selection. We have proposed a distributed protocol that jointly considers the four components and achieves weighted proportional fairness. We have also derived a greedy policy based on the distributed protocol that is easier to implement. Simulation results have shown that the distributed protocol outperforms state-of-the-art techniques. The total weighted throughputs achieved by the distributed protocol can be twice as large as state-of-the-art techniques. Simulation results have also shown that, while being suboptimal, the performance of the greedy policy is actually close to optimum quite often.

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