A Tractable Algorithm for Fair and Efficient Uplink Scheduling of Multi-hop WiMax Mesh Networks

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***Abstract*— The IEEE 802.16 standard, also known as WiMax, provides a mechanism for deploying high-speed wireless mesh network in metropolitan areas. In this paper, we propose an algorithm for the data sub-channel allocation, i.e., transmission scheduling, of WiMax based mesh networks. The goal is to increase spatial reuse, achieve high system throughput, and provide fair access for the subscriber stations. In contrast to the previous "hard" fairness definitions, we introduce a new fairness notion that is imposed contingent on the actual traffic demands, in such a way that a higher capacity region can be achieved. We formulate a scheduling problem whose objective is to maximize the system throughput under our fairness model. We also develop an efficient algorithm to find the optimal schedule and the accompanying resource allocation. The performance of the scheduling algorithm is evaluated through simulations.**

***Index Terms*— WiMax, mesh networks, fairness, centralized scheduling**

I. Introduction

The rapid increase in user demand for faster connection to the Web and VoIP services has spurred the development of new broadband access technologies over recent years. The IEEE 802.16 standard [1], also commonly known as WiMax, finalized in year 2004, aims at providing last-mile fixed wireless broadband access in the Metropolitan Area Network (MAN) with performance comparable to traditional cable, DSL or T1 networks [2]. Compared to wired solutions, WiMax provides more ubiquitous access with lower deployment and maintenance costs.

IEEE 802.16 operates at 10-66 GHz for Line-of-Sight (LOS), and 2-11 GHz for non-LOS connection. In the physical layer, the standard employs orthogonal frequency division multiplexing (OFDM), and supports adaptive modulation and coding depending on the channel conditions, providing a data rate up to 134 Mbps (per Base Station) in each 28 MHz channel. An IEEE 802.16 network consists of a Base Station (BS) and multiple Subscriber Stations (SSs). The BS serves as a gateway for the SSs to the external network, and each SS acts as an access point that aggregates traffic from end users in a certain geographical area. IEEE 802.16 support two modes of operation: Point-to-Multipoint (PMP) mode and mesh mode. In PMP each SS directly communicates with the BS through a single-hop link, which requires all SSs to be within clear LOS transmission range of the BS. In contrast, in the mesh mode, the SSs can communicate with the mesh BS and with each other through multi-hop routes via other SSs. The mesh topology not only extends the network coverage and increases capacity in non-LOS environments, but it also provides higher network reliability and availability when node or link failures occur, or when channel conditions are poor. In this paper, we focus on the mesh mode. An example of a WiMax based mesh network is illustrated in Fig.1(a).

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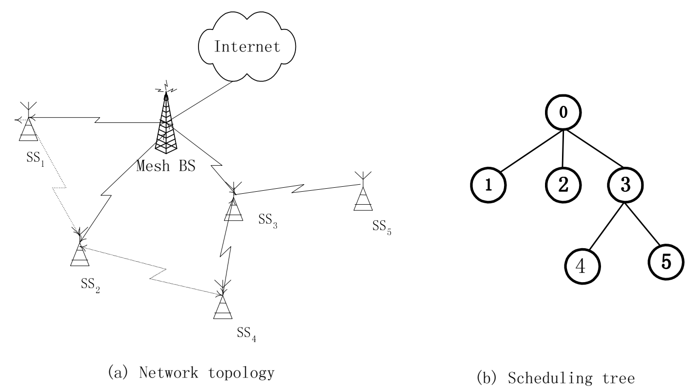

Fig. 1. An example of an IEEE 802.16 based mesh network.

The IEEE 802.16 mesh mode uses Time Division Multiple Access (TDMA) for channel access among the mesh BS and SS nodes, where a radio channel is divided into frames. Each frame is further divided into time slots that can be assigned to the BS or different SS nodes. Fig.2 shows the frame structure in the mesh mode. A frame consists of a control subframe and a data subframe. Each frame is further divided into 256 minislots for transmission of user data and control messages. In the control subframe, *transmission opportunities*, which typically consist of multiple minislots, are used to carry signalling messages for network configuration and scheduling of data subframe minislot allocation. There are two types of control subframes: network control subframe and scheduling control subframe. A network control subframe follows after every N_S scheduling control subframes, where N_S is a configurable network parameter.

In the network control subframes, *Mesh Network Configuration (MSH-NCFG)* and *Mesh Network Entry (MSH-NENT)* messages are transmitted for creation and maintenance of the network configuration. A *scheduling tree* rooted at the mesh BS is established for the routing path between each SS and the mesh BS. Active nodes within the mesh network periodically advertise *MSH-NCFG* messages which contain a *Network De-*

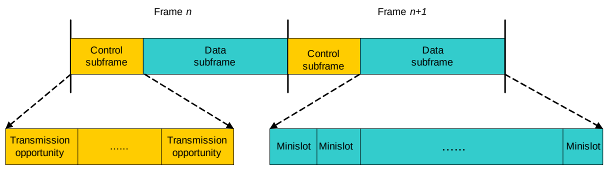

Fig. 2. The IEEE 802.16 mesh frame structure

scriptor that includes the network configuration information. A new node that wishes to join the mesh network scans for active networks by listening to *MSH-NCFG* messages. Upon receiving the *MSH-NCFG* message, the new node establishes synchronization with the mesh network. From among all the possible neighbor nodes that advertise *MSH-NCFG*, the new node select one as its *sponsor node*. Then the new node sends a *MSH-NENT* message with registration information to the mesh BS through the sponsor node. Upon receipt of the registration message, the mesh BS adds the new node as the child of the sponsor node in the scheduling tree, and then broadcasts the updated network configuration to all SSs. Fig.1(b) shows an example of the scheduling tree for the topology in Fig.1(a).

In the IEEE 802.16 mesh mode, both centralized scheduling and distributed scheduling are supported. *Mesh Centralized Schedule (MSH-CSCH)* and *Mesh Distributed Schedule (MSH-DSCH)* messages are exchanged in the scheduling control subframe to assign the data minislots to different stations. The number of transmission opportunities for *MSH-CSCH* and *MSH-DSCH* in each scheduling control subframe are network parameters that can be configured. Centralized scheduling is mainly used to transfer data between the mesh BS and the SSs, which corresponds to external traffic from the Internet; while distributed scheduling targets data delivery between two SSs in the same WiMax mesh network, which corresponds to intranet traffic. In the standard, the data subframe is partitioned into two parts for the two scheduling mechanisms respectively. The centralized scheduling handles both the *uplink*, where the traffic goes from the SSs to the mesh BS, and *downlink*, where the traffic goes from the mesh BS to the SSs. In the mesh mode, Time Division Duplex (TDD) is used to share the channel between the uplink and the downlink.

In distributed scheduling, all SSs are peers and they compete for transmission opportunities based on a pseudo-random election algorithm. A three-way handshaking procedure is employed to reserve minislots for transmitting data between neighboring SSs. The details of the mechanism and its analysis can be found in [3]. In centralized scheduling, the mesh BS acts as the centralized scheduler and determines the allocation of the minislots dedicated to centralized scheduling among all the stations. The time period for centralized scheduling is called *scheduling period*, which is typically a couple of frames in length. There are two stages in each scheduling period. In the first stage, the SSs send bandwidth requests using the *MSH-CSCH:Request* message to their sponsor nodes, which are routed to the mesh BS along the scheduling tree. Each SS not only send its own bandwidth request but also relays that of all its descendants in the scheduling tree. The SSs transmit *MSH-CSCH:Request* messages in such an order that the sponsor nodes always transmit after all their children. In this way, the mesh BS collects bandwidth requests from all the SSs. In the second stage, the mesh BS calculates and distributes the schedule by broadcasting the *MSH-CSCH:Grant* message, which is propagated to all the SSs along the scheduling tree. Since the dominant traffic in a WiMax mesh network is Internet traffic, we will focus on centralized scheduling in this paper.

Although the IEEE 802.16 mesh mode has defined the messages and signalling mechanisms for transmission scheduling, how the minislots are assigned to the different stations is left unspecified. Since scheduling is the key mechanism for resource allocation in WiMax mesh networks, an efficient scheduling algorithm is needed to achieve desired system performance. There are several previous works [4], [5], [6] that have contributed to this problem. In [4], the authors have proposed centralized scheduling and routing tree construction algorithms to provide per flow QoS in WiMax mesh networks. However, they assume that there is *no spatial reuse*, that is, only one of the links in the entire mesh network can be active in a minislot. However, in a multi-hop mesh network, allowing *spatial reuse*, i.e., allowing concurrent transmissions on links that are not interfering with each other, is very important to achieve high spectral efficiency and system throughput. In [5], simple heuristic scheduling and routing tree construction algorithms have been proposed to achieve efficient spectral utilization with spatial reuse, but the *fairness* among the SSs is not considered. Fairness is an essential objective for wireless mesh networks to ensure that subscribers receive acceptable shares of resources regardless the number of hops from the BS [8]. In [6], a transmission scheduling algorithm that achieves high channel efficiency and provides fair access to all the nodes is presented. However, the scheme enforces *hard fairness* without taking into account the actual bandwidth requests from different nodes. As can be seen in Section II, such hard fairness causes a reduction of the capacity region that can otherwise be achieved. Fairness in multi-hop wireless mesh networks is also studied in [8], [9]. These works are focused on IEEE 802.11 based mesh applications, and the fairness definition they use is also hard fairness. In this paper, we define a new fairness model where the notion of fairness is coupled to the actual traffic demands in such a way that the capacity region achieved is higher than that of "hard" fairness due to the multiplexing gain. We also provide an efficient centralized scheduling algorithm that maximizes the system utilization and achieves the new fairness objective that we define.

The remainder of this paper is organized as follows. In Section II we describe the system model and introduce our fairness definition. Then we propose an efficient algorithm that achieves the maximum throughput under our fairness model in Section III. Simulation results are presented in Section IV to evaluate the performance of the algorithm. Finally, Section V contains the conclusions.

II. System Description and Fairness Model

A. Network Model

We consider a WiMax mesh network that consists of a mesh Base Station (BS) and M mesh Subscriber Stations (SS). We label the BS as node 0 and the SS nodes as $j = 1, \cdots, M$. A link (i, j) exists between node i and j when they are within transmission range of each other, i.e., they are neighboring nodes. The mesh topology can be represented by a directed graph $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$, where $\mathcal{N} = \{0, 1, \cdots, M\}$, and $\mathcal{L} = \{1, 2, \cdots, L\}$ labels all the directed links. Assume that the capacity of link l is c_l *bps*. We focus on the centralized scheduling of the IEEE 802.16 mesh mode, where a scheduling tree rooted at the mesh BS is constructed for the routing path between each SS and the mesh BS, and the BS acts as the centralized scheduler that determines the transmission or reception of every SS in each minislot. Denote a scheduling tree by $T = \{n_0(h_{n_0}, p_{n_0}), \cdots, n_M(h_{n_M}, p_{n_M})\}$, where h_{n_i} is the number of hops from node n_i to the BS (n_0), and p_{n_i} is the parent (sponsor) node of n_i. The mesh BS is indexed by (0, 0). We denote the path from SS node i to the mesh BS by P_i. Let $\mathcal{L}_T$ be the set of links that belong to the scheduling tree T, and $L_T = |\mathcal{L}_T|$.

In each scheduling period, the mesh BS collects bandwidth requests from all SSs. Then the mesh BS calculates and distributes the transmission schedule to all the SSs. Assume that at the tth scheduling period, the bandwidth request from SS node i is $s_i(t)$ *bps*. The traffic demand vector is denoted by $\boldsymbol{s}(t) = (s_1(t), \cdots, s_M(t))$. The traffic demand varies at each scheduling period, and the mesh BS updates the schedule based on the traffic demand requirements. Since we are only concerned with one scheduling period for the scheduling algorithm, we drop the index t hereafter. The goal of a fair scheduling algorithm is to maximize the system throughput while meeting the traffic demands under fairness constraints.

B. Fairness Model

We introduce a fairness model that is different from the "hard" fairness definition [8]. However, we make the following assumptions as in [8]. First, the fairness we consider is at the granularity of a SS-aggregated flow. Typically, each SS in a WiMax mesh network acts as an access point that covers a building or residence area. The aggregated traffic at each SS could contain a number of TCP and UDP flows or data from mobile devices supported by the SS. Second, in the centralized scheduling, we exploit concurrent transmission to achieve efficient spectral utilization and high system throughput. That is, any two links that are not interfering with each other can be activated in the same minislot.

In order to define our fairness model, we need to first characterize the capacity region of the WiMax mesh network. This has been established in [11], and we briefly recapitulate it here in the context of uplink centralized scheduling for the WiMax mesh network. A set of links in the scheduling tree that can be concurrently activated in the same minislot is called an *activation set*. Such an activation set can be represented by an *activation vector* $\boldsymbol{a}$, which is a binary vector with L elements, where the lth element is

$$a_l = \begin{cases} 1, & \text{if link } l \text{ belongs to activation set } \boldsymbol{a}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $\mathcal{A}_T$ be the set of all possible activation vectors in a scheduling tree T. Assume that the rate that the mesh BS assigns to SS node i as x_i *bps*. Let $\boldsymbol{x} = (x_1, \cdots, x_M)$ denote the *bandwidth allocation vector*. Then the fraction of time that link l need to be activated is $\alpha_l = \frac{1}{c_l}\sum_{i:l\in P_i} x_i$. Let $\boldsymbol{\alpha} = (\alpha_1, \cdots, \alpha_L)$. We define the *capacity region* as follows:

Definition 1: The uplink capacity region for the WiMax mesh network under the scheduling tree T is defined as

$$\mathcal{C} = \{\boldsymbol{x} : \boldsymbol{\alpha} \in co(\mathcal{A}_T), \boldsymbol{x} \geq 0\}, \tag{1}$$

where $co(\mathcal{A}_T)$ is the convex hull of the activation vector set $\mathcal{A}_T$.

The boundary of the capacity region, denoted by $\partial\mathcal{C}$, which we called the *Pareto surface*, represents the tradeoffs among the rate allocation of different SSs.

We consider *weighted fairness* constraints, where each SS node i is assigned a weight f_i, which is determined according to pricing or other system-wide objectives. We call $\boldsymbol{f} = (f_1, \cdots, f_M)$ the *fairness profile*. As compared to other fairness schemes which enforce "hard" fairness, where each user is assigned a bandwidth proportional to its weight regardless of the actual traffic demand [8], we impose fairness constraints contingent on the bandwidth requests. If the traffic demand vector $\boldsymbol{s}$ is within the capacity region $\mathcal{C}$, then even though the bandwidth request s_i may not be proportional to the weight f_i, e.g., $\boldsymbol{s}$ in Fig.3, we can still meet all the traffic demands. However, when the traffic demand vector is outside the capacity region, we do need to impose fairness constraints to find an efficient rate allocation point on the Pareto surface. However, it may *not* be necessary to impose fairness constraints on *all* the SS nodes. Define the *relative bandwidth request* $R_i \triangleq \frac{s_i}{f_i}$. When the relative traffic demands of some SSs are low, we may still meet their requests. Thus, we only impose the constraints on those SSs whose demands cannot be met without violating the fairness constraints relative to other SS nodes.

Suppose that the set of SSs on which we impose fairness constraints is denoted by $\mathcal{I}$. Then we have

$$\frac{x_i}{f_i} = R, \quad x_i \leq s_i, \quad i \in \mathcal{I}; \tag{2}$$

and for the other SSs whose demands are satisfied, we must have

$$x_j = s_j, \quad \frac{s_j}{f_j} \leq R, \quad j \in \mathcal{I}^c = \mathcal{N} \setminus \mathcal{I}. \tag{3}$$

Now we can give a formal mathematical definition of our fairness model by writing the above constraints in a concise form:

*Definition 2: Given the fairness profile $\boldsymbol{f}$ and uplink traffic demand $\boldsymbol{s}$ of a WiMax mesh network, a rate allocation vector $\boldsymbol{x}$ is **fair** contingent on their traffic demand, if the following condition is satisfied:*

$$x_i = \min\{s_i, f_i R\}, \quad i \in \mathcal{N}. \tag{4}$$

Our goal is to get maximum throughput under the above fairness definition, subject to the capacity constraint. Hence our goal is to find a rate allocation vector $\boldsymbol{x}$ in the capacity region that achieves the maximum R such that (4) is satisfied.

This can be formulated as the *optimal fair rate allocation* (OFRA) problem:

$$\max \quad R \tag{5}$$
$$\text{s.t.} \quad x_i = \min\{s_i, f_i R\}, \quad i \in \mathcal{N}, \tag{6}$$
$$\boldsymbol{x} \in \mathcal{C}, \tag{7}$$

where $\mathcal{C}$ is the capacity region defined in (1). In the next section, we will describe an efficient fair scheduling algorithm that solves the above problem by exploiting the characteristics of WiMax mesh network.

First we further illustrate the definition graphically in Fig.3. Suppose that at a scheduling period, the traffic demand is $\boldsymbol{s}'$, as indicated in Fig.3, where the relative traffic demand $\frac{s_2}{f_2}$ is low. In this case, the optimal fair allocation is $\boldsymbol{x}'$, where we can indeed meet the demand of s_1', and find the maximizing $x_2' = R'$ on the Pareto surface. Here the condition $\frac{s_1'}{f_1} \le R'$ must be satisfied. As an example of the other case, suppose the traffic demand is $\boldsymbol{s}''$. Then we cannot meet the demand of either s_1'' or s_2'' without violating the constraints (4). Then the optimal allocation is $\boldsymbol{x}''$, where we maximize R'' such that $\frac{x_1''}{f_1} = \frac{x_2''}{f_2} = R''$ on the Pareto surface. With our fairness model, when some of the SSs have low traffic load, the capacity can be allocated to other SSs with high traffic load. As the traffic loads of all SSs vary at different scheduling periods, a *multiplexing gain* can be achieved while fairness is still ensured. Note that if instead one imposes "hard" fairness constraints, then the actual achievable capacity region is $\mathcal{C}'$ instead of $\mathcal{C}$.

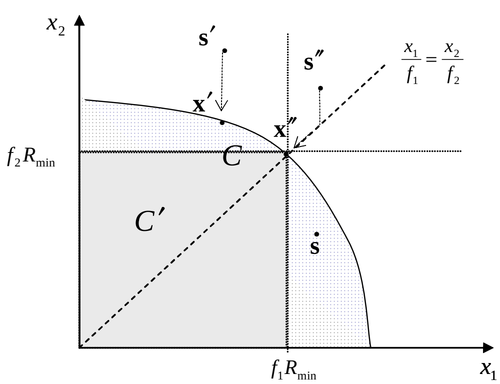

Fig. 3. Illustration of the fairness model.

III. Fair Scheduling for WiMax Mesh Networks

In this section, we develop an uplink fair scheduling algorithm for WiMax mesh networks, according to the fairness model described above. We need to find an efficient algorithm to solve the OFRA problem (5). First we observe that the difficulties in solving (5) are:

1) The fairness condition (4) *cannot* be replaced by a set of linear constraints. It is only when the set $\mathcal{I}$ is fixed that (4) is equivalent to combining (2) and (3), and one gets a *reduced optimal fair rate allocation* (ROFRA) optimization problem:

$$\max \quad R \tag{8}$$
$$\text{s.t.} \quad x_i = f_i R, \quad x_i \le s_i, \quad i \in \mathcal{I}, \tag{9}$$
$$x_j = s_j, \quad s_j \le f_j R, \quad j \in \mathcal{I}^c, \tag{10}$$
$$\boldsymbol{x} \in \mathcal{C}. \tag{11}$$

However, the number of all possible sets $\mathcal{I}$ is exponential in the number of SS nodes M.

2) Even for the ROFRA problem, since the number of possible activation sets that define the capacity region is typically exponential in the number of links L, it is still hard to solve it efficiently.

In this section, we will show that the OFRA problem can be solved by solving a sequence of (less than M) ROFRA problems. Additionally, by exploring the structure and characteristics of WiMax mesh network, we further reduce the ROFRA problem to a simple linear programming (LP) problem. Before developing our solution, we first establish the following lemma.

Lemma 1: Consider the ROFRA problem (8) with the set $\mathcal{I}$ of SSs on which fairness constraints are imposed. Let $R_{min}^{\mathcal{I}} = \min_{i\in\mathcal{I}} R_i$, and $R_{max}^{\mathcal{I}^c} = \max_{j\in\mathcal{I}^c} R_j$. If the optimal solution R^* exists, it must satisfy

$$R_{max}^{\mathcal{I}^c} \le R^* \le R_{min}^{\mathcal{I}}; \tag{12}$$

otherwise the problem is infeasible.

Proof: For any R in the feasible region of problem (8), from (9) we know $f_i R \le s_i$, or $R \le \frac{s_i}{f_i} = R_i, \forall i \in \mathcal{I}$, which implies $R \le R_{min}^{\mathcal{I}} = \min_{i\in\mathcal{I}} R_i$. From (10) we have $R \ge \frac{s_j}{f_j} = R_j, \forall j \in \mathcal{I}^c$, which implies $R \ge R_{max}^{\mathcal{I}^c} = \max_{j\in\mathcal{I}^c} R_j$. So if the optimal solution R^* exists, it must be within the interval $[R_{max}^{\mathcal{I}^c}, R_{min}^{\mathcal{I}}]$. Otherwise, if $R_{max}^{\mathcal{I}^c} > R_{min}^{\mathcal{I}}$, then the problem (8) is infeasible. □

From Lemma 1 we know that the set $\mathcal{I}$ must be chosen such that the relative bandwidth requests of all the nodes in the set $\mathcal{I}^c$ are less than or equal to that of any of the nodes in the set $\mathcal{I}$. Otherwise there is no feasible solution. We therefore order the relative bandwidth requests of all the SSs, from lowest to the highest. Without loss of generality, we can assume $R_1 \le R_2 \le \cdots \le R_M$. Then there are only $M+1$ possible choices of such sets: $\emptyset$, $\{M\}$, $\{M-1, M\}, \ldots, \{1, 2, \cdots, M\}$. Note that: (i) the problem (8) may still be infeasible with the above chosen $M+1$ sets, except for $\mathcal{I} = \{1, 2, \cdots, M\}$, due to the capacity constraint (11); (ii) the optimal solution of (5) is the maximum of the optimal solutions of (8) under the above $M+1$ possible chosen sets $\mathcal{I}$. Now we can describe the *fair uplink scheduling* (FUS) algorithm as follows.

Algorithm 1: Fair Uplink Scheduling (FUS)

If $\boldsymbol{s} \in \mathcal{C}$
    Let $\boldsymbol{x} = \boldsymbol{s}$, no fairness constraints imposed
    Return
Else
    $k = M$
    While $k \geq 1$
        Let $\mathcal{I} = \{k, \cdots, M\}$, solve the ROFRA problem
        If feasible with optimal solution $\{R^*, \boldsymbol{x}^*\}$
            Let $\boldsymbol{x} = \boldsymbol{x}^*$, return
        Else
            k = k -1
        End
    End while
End

Note that in this algorithm, the condition $\boldsymbol{s} \in \mathcal{C}$ in the first line in fact corresponds to the case of solving (8) with $\mathcal{I} = \emptyset$. The following theorem directly follows from Lemma 1 and the above arguments.

Theorem 1: The fair uplink scheduling (FUS) algorithm solves the the OFRA problem (5) and finds the optimal rate allocation vector $\boldsymbol{x}$ under our fairness model.

Now we have reduced the OFRA problem to solving a sequence of ROFRA problems. However, for general mesh network, solving the ROFRA problem still falls into the category of NP hard problem due to the existence of an exponential number of activation sets. These kinds of problems have been well studied in [11]. In contrast, in this work, by exploring the structure and characteristics of WiMax mesh networks, we transform the ROFRA problem to a linear programming formulation, which can be solved efficiently. Note that the link activation constraint arises due to conflicts and interference among the links. There are several types of constraints in wireless mesh networks:

1) Due to the half duplex nature of the transceiver, a node cannot transmit and receive simultaneously.
2) A node cannot transmit to multiple neighbor nodes at the same time, or receive from multiple neighbors at the same time.
3) The transmission of one link can be corrupted by the interference from a neighboring link.

We call the first two types of constraints *primary conflicts*, and the third constraint as a *secondary conflict* which depends on the physical layer parameters and capabilities. Note that in WiMax mesh network, the BS and SSs are typically equipped with *directional* (e.g. beamforming) *antennas* which can concentrate their transmit energy in the direction of the intended receiver while minimizing the interference caused to neighboring links. By carefully planning the locations of BS and SS nodes, the interference among neighboring links can be greatly reduced. So we assume that only primary conflicts need to be considered for link activation, which requires that only one incoming or outgoing link can be active for each node at any time. Let $\mathcal{N}(i)$ be the set of links that are incoming to or outgoing from node i, $\boldsymbol{x}$ be the bandwidth allocation vector, and $\alpha_l = \frac{1}{c_l}\sum_{i:l\in P_i} x_i$ be the fraction of time that link l needs to be activated. Then a necessary condition for $\boldsymbol{x} \in \mathcal{C}$ is:

$$\sum_{l:l\in\mathcal{N}(i)} \alpha_l \leq 1, \quad \forall i \in \mathcal{N}. \tag{13}$$

In the following lemma, we show that (13) is also a sufficient condition for $\boldsymbol{x} \in \mathcal{C}$, by exploring the tree structure of the uplink WiMax mesh network.

Lemma 2: Assuming that only primary conflicts exist in WiMax mesh network, the necessary and sufficient condition that a bandwidth allocation vector $\boldsymbol{x}$ is schedulable, i.e., $\boldsymbol{x} \in \mathcal{C}$, is (13).

Proof: The fact that (13) is a necessary condition follows directly from the definition of primary constraints.

To prove that it is also sufficient, we construct the scheduling multi-graph as in [10]. Let T_S be the length of the scheduling period, σ the minislot time, and $K = \frac{T_S}{\sigma}$ the number of minislots in a scheduling period. For convenience, assume that all the minislots are dedicated for uplink centralized scheduling, and that we can choose K large enough such that the number of minislots that link l needs to be activated in a scheduling period $w_l = \alpha_l K$ is an integer for every $l \in \mathcal{L}_T$. Then we construct the multi-graph scheduling tree $T_m(\boldsymbol{x})$ corresponding to the scheduling tree T. $T_m(\boldsymbol{x})$ has the same node set $\mathcal{N}$ as T, with a link $l \in \mathcal{L}_T$ represented by w_l edges in $T_m(\boldsymbol{x})$ between the same endpoint nodes. An example of the construction of the multi-graph scheduling tree is illustrated in Fig.4.

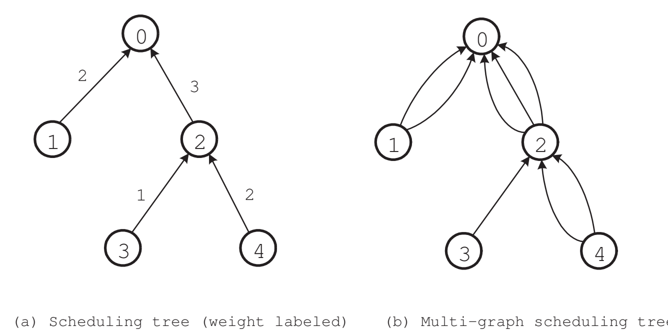

Fig. 4. Illustration of the construction of the multi-graph scheduling tree.

The number minislots that we need to meet the traffic demand vector $\boldsymbol{x}$, is equal to the *chromatic index* Γ of the multi-graph scheduling tree $T_m(\boldsymbol{x})$, which is the minimum number of colors needed to color the edges of the multi-graph, such that no two edges incident on the same node are assigned the same color. From graph theory we know that Γ is equal to the maximum of the cardinalities of all the *cliques* of the complementary graph of the multi-graph. Since we assume only primary conflicts exist, any pair of edges in a clique of the complementary graph must be incident to or from a common node. Each clique is a maximal set of edges where any pair of them is incident on a common node. It is obvious that in a multi-graph tree, each clique corresponds to one of the nodes, and is composed of all the links incident on that node. So we have $\Gamma = \max_{i\in\mathcal{N}} \sum_{l\in\mathcal{N}(i)} w_l \triangleq \Delta$, which is the maximum degree of a node in the multi-graph scheduling tree $T_m(\boldsymbol{x})$. Note that (13) implies that $\sum_{l:l\in\mathcal{N}(i)} w_l = \sum_{l:l\in\mathcal{N}(i)} \alpha_l K \leq K,\ \forall i \in \mathcal{N}$, from which we have $\Gamma \leq K$. This means that we can schedule the traffic demand vector $\boldsymbol{x}$ in a scheduling period. This shows that (13) is a sufficient condition for the schedulability of $\boldsymbol{x}$. □

With Lemma 2 in hand we can reduce the ROFRA problem (8) to a simple *linear programming problem*, which we state formally in the following theorem.

Theorem 2: Assume that only primary conflicts exist in WiMax mesh network, then the ROFRA problem (8) is equivalent to the following linear programming (ROFRA-LP) problem:

$$\max \quad R \tag{14}$$

$$\text{s.t.} \quad x_i = f_i R, \quad x_i \leq s_i, \quad i \in \mathcal{I}, \tag{15}$$

$$x_j = s_j, \quad s_j \leq f_j R, \quad j \in \mathcal{I}^c, \tag{16}$$

$$\alpha_l \geq \frac{1}{c_l} \sum_{i:l\in P_i} x_i, \quad l \in \mathcal{L}_T, \tag{17}$$

$$\sum_{l:l\in\mathcal{N}(i)} \alpha_l \leq 1, \quad \forall i \in \mathcal{N}. \tag{18}$$

In the above discussion, we have assumed that the number of minislots in a scheduling period K can be chosen arbitrarily large to make all $w_l = \alpha_l K$ integers. Now we take this granularity issue into account. Assume that K_C^u slots are dedicated to uplink centralized scheduling in each scheduling period, which is fixed, w_l is the number of slots that are allocated to link l in the scheduling period, and denote $\boldsymbol{w} = (w_1, w_2, \cdots, w_L)$. The constraints (17) and (18) in ROFRA-LP can be rewritten as:

$$\frac{w_l \sigma}{T_S} \geq \frac{1}{c_l} \sum_{i:l\in P_i} x_i, \quad l \in \mathcal{L}_T, \tag{19}$$

$$\sum_{l:l\in\mathcal{N}(i)} w_l \leq K_C^u, \quad \forall i \in \mathcal{N}. \tag{20}$$

Since the ROFRA-LP problem is a LP, it can be computationally solved very efficiently. By replacing the ROFRA in the FUS algorithm with the above ROFRA-LP, we can solve a sequence of LPs to find the optimal rate allocation vector $\boldsymbol{x}$, along with the optimal slot allocation vector $\boldsymbol{w}$, according to the request $\boldsymbol{s}$ and fairness profile $\boldsymbol{f}$. Furthermore, one can notice that the multi-graph tree constructed in the proof of Lemma 2 can be greedily colored to generate the schedule, once $\boldsymbol{w}$ is feasible. So, once we find the optimal $\boldsymbol{w}$, we can actually construct a minislot allocation schedule by greedily coloring the multi-graph scheduling tree.

IV. Simulation Results

In this section, we evaluate the performance of the proposed algorithm through simulations. We consider a WiMax mesh network with 10 SS nodes with the scheduling tree topology shown in Fig.5. We conduct simulations for centralized scheduling of uplink traffic only. The scheduling for downlink traffic can be performed and analyzed similarly.

TABLE I

Mesh network parameters in the simulation

Frame duration (T_F)	10 ms
No. of OFDM symbols / frame	1024
No. of OFDM symbols / minislot (n_s)	4
No. of minislots / frame (N)	256
No. of minislots / frame for uplink CSCH (N_C)	200
No. of bytes / OFDM symbol (B)	72
Scheduling period (T_S)	100 ms

The physical and MAC layer parameters of the WiMax mesh network used in the simulation are summarized in Table I. Since in WiMax mesh networks, the SSs typically have directional antennas fixed on top of buildings with LOS connections between each other, we assume the channel condition is static, with constant burst rate B. For convenience, we choose the link capacity of all the links in the mesh network to be the same, which is

$$c = c_l = \frac{8 n_s B}{T_F/N} \approx 59Mbps, \quad l \in \mathcal{L}. \tag{21}$$

We adopt the Bernoulli arrival traffic model for the simulation. The traffic is randomly generated by each SS node i at every minislot with the amount of data uniformly distributed in $[0, 2s_i\frac{T_F}{N}]$, which corresponds a mean traffic arrival rate s_i. For convenience, we assume that all the SSs have the same mean traffic demand, i.e., $s_i = s, i \in \mathcal{N}$. In each scheduling period, the mesh BS collects the aggregated traffic demands of all the SSs and calculates the schedule according to the scheduling algorithm. We choose the scheduling period to be 10 frames. The simulation time is set to be 100 seconds.

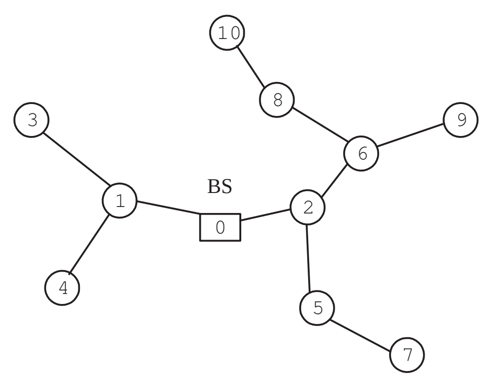

Fig. 5. Network topology used in the simulations.

For comparison, we also implement the following scheduling schemes, in addition to our *fair uplink scheduling* (FUS) algorithm:

1) *Hard fair scheduling with spatial reuse* (HF-SR). Each SS node i is assigned a bandwidth which is proportional to its weight f_i regardless of its traffic demand $s_i(t)$ at each scheduling period t. Concurrent transmission is allowed in the scheduling.
2) *Hard fair scheduling without spatial reuse* (HF-NSR). The scheduling is the same as HF-SR except that concurrent transmissions are not allowed in the scheduling.

3) *Admission control only* (AC). The traffic demands of the users with larger $s_i(t)$'s are denied (set to 0) until $\boldsymbol{s}$ is admissible. Spatial reuse is allowed in the scheduling. This corresponds to the admission control scheme without a fairness guarantee.

In the first setup, we choose the fairness weights to be the same, i.e., $f_i = 1$, $i \in \mathcal{N}$; and run the simulation under different mean traffic demands s. We compare the total achieved throughput of the mesh network $S = \sum_{i\in\mathcal{N}} x_i$ under the four schemes. The results are plotted in Fig.6, where both the total throughput S and mean traffic demand s are normalized, i.e., divided by c. From the results we can see that our fair scheduling algorithm is strictly better than other three schemes, providing a maximum improvement of 23.2% over the HF-SR scheme. This is due to the multiplexing gain exploited by the FUS scheme. In contrast, the HF-SR achieves a bandwidth efficiency improvement of 72% on average over HF-NSR by allowing spatial reuse. To demonstrate the proposed algorithm's fairness provisioning, Fig.7 depicts the per SS achieved throughput at the saturated traffic demand $s = 0.3c$. The results show that the FUS algorithm provides fair throughput to all the SSs with a standard deviation of 0.04%. HF-SR also guarantees fairness but the achieved throughputs for some SSs are lower than the AC scheme; while our FUS scheme achieves strictly higher throughput than the AC and ensuress fairness at the same time.

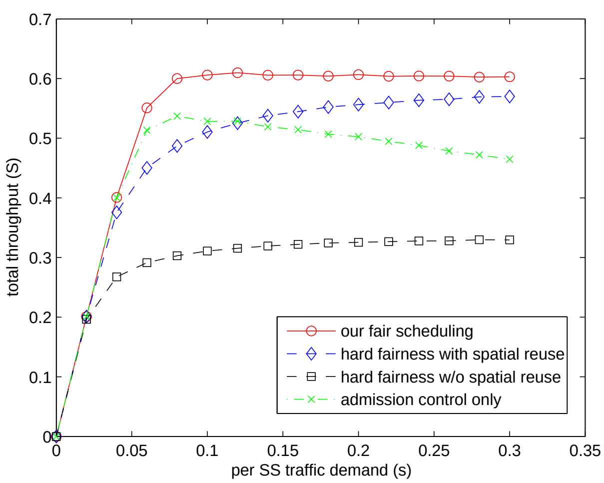

Fig. 6. Total throughput for $\boldsymbol{f} = (1, 1, \cdots, 1)$.

To demonstrate the case where the SS nodes have different fairness weights, we consider the case where $\boldsymbol{f} = (1, 1, 1, 1, 1, 2, 2, 2, 2, 2)$ and run the simulation. The results for the total throughput are shown in Fig.8, and the results of per SS throughputs are shown in Fig.9. The FUS achieves a maximum total throughput improvement of 35.4% over HF-SR, which shows that the multiplexing gain is larger when the fairness weights are unequal. However, when the traffic demands become saturated, the multiplexing gain diminishes. From both Fig.7 and Fig.9 we can see that when there is admission control only, the SS nodes that are fewer hops away from the mesh BS get a larger share of the bandwidth,

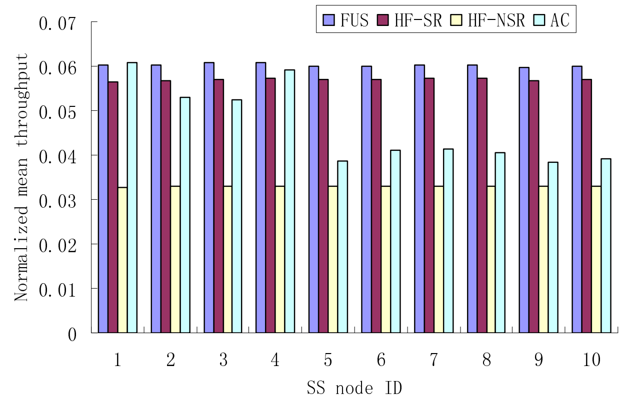

Fig. 7. Per SS throughput for $\boldsymbol{f} = (1, 1, \cdots, 1)$.

while the SSs that are further away are assigned less bandwidth, which is the well-known *spatial bias* phenomenon [8]. With our FUS scheme, all the SS nodes get shares that are proportional to their fairness weights, i.e., the spatial bias is eliminated.

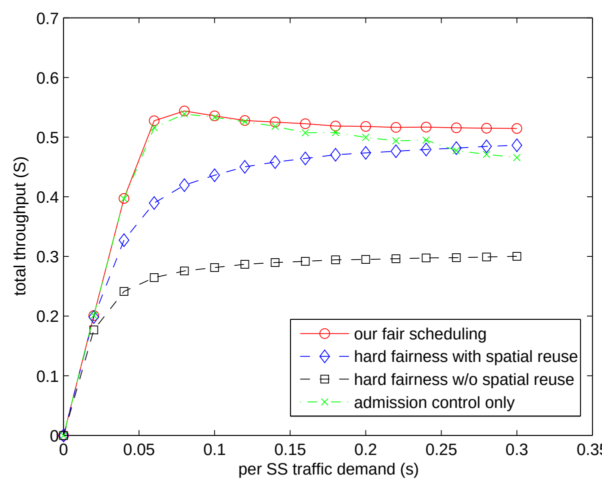

Fig. 8. Total throughput for $\boldsymbol{f} = (1, 1, 1, 1, 1, 2, 2, 2, 2, 2)$.

V. Concluding Remarks

In this paper, we have presented an efficient fair scheduling algorithm for IEEE 802.16 multi-hop mesh networks according to a new fairness model that we have defined. In this new fairness model, the bandwidth allocation is contingent on the actual traffic demands in such a way that the capacity region is not sacrificed by imposing the fairness constraints. We formulate the scheduling problem as one of maximizing the system throughput subject to the fairness notion. By exploiting the characteristics of WiMax mesh networks, we have developed an efficient algorithm to find the optimal schedule corresponding to the optimum of the formulated problem. The simulation results indicate that our scheme achieves higher system throughput than "hard" fairness schemes, while ensuring fairness among the SSs in the WiMax mesh networks,

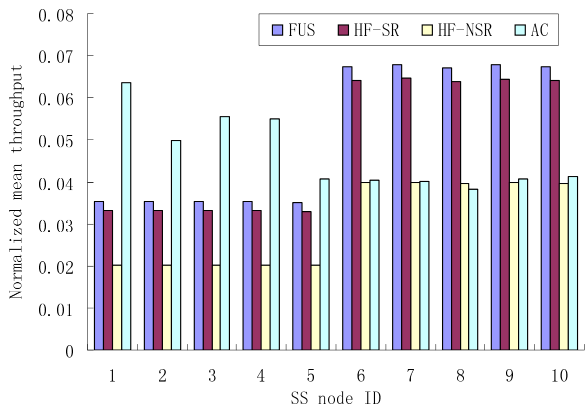

Fig. 9. Per SS throughput for $\boldsymbol{f} = ((1, 1, 1, 1, 1, 2, 2, 2, 2, 2)$.

especially when the the number of hops is large and the SS nodes have uneven fairness weights. Our scheme provides an example of how to enhance fairness in multi-hop wireless networks.

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