Utility Maximization for Delay Constrained QoS in Wireless

I-Hong Hou
CSL and Department of CS
University of Illinois
Urbana, IL 61801, USA
ihou2@illinois.edu

P. R. Kumar
CSL and Department of ECE
University of Illinois
Urbana, IL 61801, USA
prkumar@illinois.edu

***Abstract*—This paper studies the problem of utility maximization for clients with delay based QoS requirements in wireless networks. We adopt a model used in a previous work that characterizes the QoS requirements of clients by their delay constraints, channel reliabilities, and timely throughput requirements. In this work, we assume that the *utility* of a client is a function of the timely throughput it obtains. We treat the timely throughput for a client as a tunable parameter by the access point (AP), instead of a given value as in the previous work. We then study how the AP should assign timely throughputs to clients so that the total utility of all clients is maximized.**

We apply the techniques introduced in two previous papers to decompose the utility maximization problem into two simpler problems, a $CLIENT$ problem and an $ACCESS$-$POINT$ problem. We show that this decomposition actually describes a bidding game, where clients bid for the service time from the AP. We prove that although all clients behave selfishly in this game, the resulting equilibrium point of the game maximizes the total utility. In addition, we also establish an efficient scheduling policy for the AP to reach the optimal point of the $ACCESS$-$POINT$ problem. We prove that the policy not only approaches the optimal point but also achieves some forms of fairness among clients. Finally, simulation results show that our proposed policy does achieve higher utility than all other compared policies.

I. Introduction

We study how to provide QoS to maximize utility for wireless clients. We jointly consider the delay constraint and channel unreliability of each client. The access point (AP) assigns timely throughputs to clients under the delay and reliability constraints. This distinguishes our work from most other work on providing QoS where the timely throughputs to clients are taken as given inputs rather than tunable parameters.

We consider the scenario where there is one AP that serves a set of wireless clients. We extend the model proposed in a previous work [8]. This model analytically describes three important factors for QoS: delay, channel unreliability, and timely throughput. The previous work also provides a necessary and sufficient condition for the demands of the set of clients to be feasible. In this work, we treat the timely throughputs for clients as variables to be determined by the AP. We assume that each client receives a certain amount of *utility* when it is provided a timely throughput. The relation between utility and timely throughput is described by an *utility function*, which may differ from client to client. Based on this model, we study the problem of maximizing the total utility of all clients, under feasibility constraints. We show that this problem can be formulated as a convex optimization problem.

Instead of solving the problem directly, we apply the techniques introduced by Kelly [10] and Kelly, Maulloo, and Tan [11] to decompose the problem of system utility maximization into two simpler subproblems that describe the behaviors of the clients and the AP, respectively. We prove that the utility maximization problem can be solved by jointly solving the two simpler subproblems. Further, we describe a bidding game for the reconciliation between the two subproblems. In this game, clients bid for service time from the AP, and the AP assigns timely throughputs to clients according to their bids, to optimize its own subproblem, under feasibility constraints. Based on the AP's behavior, each client aims to maximize its own net utility, that is, the difference between the utility it obtains and the bid it pays. We show that, while all clients behave selfishly in the game, the equilibrium point of the game solves the two subproblems jointly, and hence maximizes the total utility of the system.

We then address how to design a scheduling policy for the AP to solve its subproblem. We propose a very simple priority based scheduling algorithm for the AP. This policy requires no information of the underlying channel qualities of the clients and thus needs no overhead to probe or estimate the channels. We prove that the long-term average performance of this policy converges to a single point, which is in fact the solution to the subproblem for the AP. Further, we also establish that the policy achieves some forms of fairness.

Our contribution is therefore threefold. First, we formulate the problem of system utility maximization as a convex optimization problem. We then show that this problem is amenable to solution by a bidding game. Finally, we propose a very simple priority based AP scheduling policy to solve the AP's subproblem, that can be used in the bidding iteration to reach the optimal point of the system's utility maximization problem.

Finally, we conduct simulation studies to verify all the theoretical results. Simulations show that the per-

This material is based upon work partially supported by USARO under Contract Nos. W911NF-08-1-0238 and W-911-NF-0710287, AFOSR under Contract FA9550-09-0121, and NSF under Contract Nos. CNS-07-21992, ECCS-0701604, CNS-0626584, and CNS-05-19535.

formance of the proposed scheduling policy converges quickly to the optimal value of the subproblem for AP. Also, by jointly applying the scheduling policy and the bidding game, we can achieve higher total utility than all other compared policies.

The rest of the paper is organized as follows: Section II reviews some existing related work. Section III introduces the model for QoS proposed in [8] and also summarizes some related results. In Section IV, we formulate the problem of utility maximization as a convex programming problem. We also show that this problem can be decomposed into two subproblems. Section V describes a bidding game that jointly solves the two subproblems. One phase of the bidding game consists of each client selfishly maximizing its own net profit, and the other phase consists of the AP scheduling client transmissions to optimize its subproblem. Section VI addresses the scheduling policy to optimize this latter subproblem. Section VII demonstrates some simulation studies. Finally, Section VIII concludes this paper.

II. Related Work

There has been a lot of research on providing QoS over wireless channels. Most of the research has focused on admission control and scheduling policies. Hou, Borkar, and Kumar [8] and Hou and Kumar [9] have proposed analytical models to characterize QoS requirements, and have also proposed both admission control and scheduling policies. Ni, Romdhani, and Turletti [13] provides an overview of the IEEE 802.11 mechanisms and discusses the limitations and challenges in providing QoS in 802.11. Gao, Cai, and Ngan [7], Niyato and Hossain [14], and Ahmed [1] have surveyed existing admission control algorithms in different types of wireless networks. On the other hand, Fattah and Leung [6] and Cao and Li [5] have provided extensive surveys on scheduling policies for providing QoS.

There is also research on utility maximization for both wireline and wireless networks. Kelly [10] and Kelly, Maulloo, and Tan [11] have considered the rate control algorithm to achieve maximum utility in a wireline network. Lin and Shroff [12] has studied the same problem with multi-path routing. As for wireless networks, Xiao, Shroff, and Chong [15] has proposed a power-control framework to maximize utility, which is defined as a function of the signal-to-interference ratio and cannot reflect channel unreliability. Cao and Li [4] has proposed a bandwidth allocation policy that also considers channel degradation. Bianchi, Campbell, and Liao [2] has studied utility-fair services in wireless networks. However, all the aforementioned works assume that the utility is only determined by the allocated bandwidth. Thus, they do not consider applications that require delay bounds.

III. System Model and Feasibility Condition

We adopt the model proposed in a previous work [8] to capture two key QoS requirements, delay constraints and timely throughput requirements, and incorporating channel conditions for users. In this section, we describe the proposed model and summarize relevant results of [8].

We consider a system with N clients, numbered as $\{1, 2, \dots, N\}$, and one access point (AP). Packets for clients arrive at the AP and the AP needs to dispatch packets to clients to meet their respective requirements. We assume that time is slotted, with slots numbered as $\{0, 1, 2, \dots\}$. The AP can make exactly one transmission in each time slot. Thus, the length of a time slot would include the times needed for transmitting a DATA packet, an ACK, and possibly other MAC headers. Assume there is one packet arriving at the AP periodically for each client, with a fixed period of τ time slots, at time slots $0, \tau, 2\tau, \dots$. Each packet that arrives at the beginning of a period $[k\tau, (k+1)\tau)$ must be delivered within the ensuing period, or else it expires and is dropped from the system at the end of this period. Thus, a delay constraint of τ time slots is enforced on all successfully delivered packet. Further, unreliable and heterogeneous wireless channels to these clients are considered. When the AP makes a transmission for client n, the transmission succeeds (by which is meant the successful deliveries of both the DATA packet and the ACK) with probability p_n. Due to the unreliable channels and delay constraint, it may not be possible to deliver the arrived packets of all the clients. Therefore, each client stipulates a certain timely throughput q_n that it has to receive, which is defined as the average number of successfully delivered packets for client n per period. The previous work also shows how this model can be used to capture scenarios where both uplink traffic and downlink traffic exist.

Below we describe the formal definitions of the concepts of *fulfilling* a set of clients and the *feasibility* of a set of client requirements.

Definition 1: A set of clients with the above QoS constraints is said to be *fulfilled* by a particular scheduling policy η of the AP if the time averaged timely throughput of each client is at least q_n with probability 1.

Definition 2: A set of clients is *feasible* if there exists some scheduling policy of the AP that fulfills it.

Whether a certain client is fulfilled can be decided by the average number of time slots that the AP spends on working for the client per period:

Lemma 1: The timely throughput of client n converges to q_n with probability one if and only if the *work performed on client* n, defined as the long-term average number of time slots that the AP spends on working for client n per period, converges to $w_n(q_n) = \frac{q_n}{p_n}$ with probability one. We therefore call $w_n(q_n)$ the *workload* of client n.

Since expired packets are dropped from the system at the end of each period, there is exactly one packet for each client at the beginning of each period. Therefore, there may be occasions where the AP has delivered all packets before the end of a period and is therefore forced to stay idle for the remaining time slots in the period. Let I_S be the expected number of such forced idle time slots in a period when the client set is just $S \subseteq \{1, 2, \dots, N\}$ (i.e., all clients except those in S are removed from consideration), and the AP only caters to the subset S of clients. Since each client $n \in S$ requires w_n time slots per period on average, we can obtain a necessary condition for feasibility: $\sum_{i\in S} w_i(q_i) + I_S \le \tau$, for all $S \subseteq \{1, 2, \ldots, N\}$. It is shown in [8] that this necessary condition is also sufficient:

Theorem 1: A set of clients, with timely throughput requirements $[q_n]$, is feasible if and only if $\sum_{i\in S} \frac{q_i}{p_i} \le \tau - I_S$, for all $S \subseteq \{1, 2, \ldots, N\}$.

IV. Utility Maximization and Decomposition

In the previous section, it is assumed that the timely throughput requirements, $[q_n]$, are given and fixed. In this paper, we address the problem of how to choose $q := [q_n]$ so that the total utility of all the clients in the system can be maximized.

We begin by supposing that each client has a certain utility function, $U_n(q_n)$, which is strictly increasing, strictly concave, and continuously differentiable function over the range $0 < q_n \le 1$, with the value at 0 defined as the right limit, possibly $-\infty$. The problem of choosing q_n to maximize the total utility, under the feasibility constraint of Theorem 1, can be described by the following convex optimization problem:

SYSTEM:

$$\text{Max } \sum_{i=1}^N U_i(q_i) \tag{1}$$
$$\text{s.t. } \sum_{i\in S} \frac{q_i}{p_i} \le \tau - I_S, \forall S \subseteq \{1, 2, \ldots, N\}, \tag{2}$$
$$\text{over } q_n \ge 0, \forall 1 \le n \le N. \tag{3}$$

It may be difficult to solve $SYSTEM$ directly due to the following two reasons. First, the utility functions can vary from client to client and be known only to the client. Second, there are exponentially many feasibility constraints. Thus, for example, a dual decomposition solution is intractable. So, we decompose it into two simpler problems, namely, $CLIENT$ and $ACCESS\text{-}POINT$, as described below. This decomposition was first introduced by Kelly [10], though in the context of dealing with rate control for non-real time traffic. Though the $ACCESS\text{-}POINT$ still involves exponentially many constraints, we will see in Section VI that there exists a simple scheduling policy for this subproblem, which makes this an attractive approach.

Suppose client n is willing to pay an amount of ρ_n per period, and receives a long-term average timely throughput q_n proportional to ρ_n, with $\rho_n = \psi_n q_n$. If $\psi_n > 0$, the utility maximization problem for client n is:

CLIENT$_n$:

$$\text{Max } U_n(\frac{\rho_n}{\psi_n}) - \rho_n \tag{4}$$
$$\text{over } 0 \le \rho_n \le \psi_n. \tag{5}$$

On the other hand, given that client n is willing to pay ρ_n per period, we suppose that the AP wishes to find the vector q to maximize $\sum_{i=1}^N \rho_i \log q_i$, under the feasibility constraints. In other words, the AP has to solve the following optimization problem:

ACCESS-POINT:

$$\text{Max } \sum_{i=1}^N \rho_i \log q_i \tag{6}$$
$$\text{s.t. } \sum_{i\in S} \frac{q_i}{p_i} \le \tau - I_S, \forall S \subseteq \{1, 2, \ldots, N\}, \tag{7}$$
$$\text{over } q_n \ge 0, \forall 1 \le n \le N. \tag{8}$$

We begin by showing that solving $ACCESS\text{-}POINT$ is equivalent to jointly solving $CLIENT_n$ and $ACCESS\text{-}POINT$.

Theorem 2: There exist non-negative vectors q, $\rho := [\rho_n]$, and $\psi := [\psi_n]$, with $\rho_n = \psi_n q_n$, such that:

(i) For n such that $\psi_n > 0$, ρ_n is a solution to $CLIENT_n$;
(ii) Given that each client n pays ρ_n per period, q is a solution to $ACCESS\text{-}POINT$.

Further, if q, ρ, and ψ are all positive vectors, the vector q is also a solution to $SYSTEM$.

Proof: We will first show the existence of q, ρ, and ψ that satisfy (i) and (ii). We will then show that the resulting q is also the solution to $SYSTEM$.

There exists some $\epsilon > 0$ so that by letting $q_n \equiv \epsilon$, the vector q is an interior point of the feasible region for both $SYSTEM$ (2) (3), and $ACCESS\text{-}POINT$ (7) (8). Also, by setting $\rho_n \equiv \epsilon$, ρ_n is also an interior point of the feasible region for $CLIENT_n$ (5). Therefore, by Slater's condition, a feasible point for $SYSTEM$, $CLIENT_n$, or $ACCESS\text{-}POINT$, is the optimal solution for the respective problem if and only if it satisfies the corresponding Karush-Kuhn-Tucker (KKT) condition for the problem. Further, since the feasible region for each of the problems is compact, and the utilities are continuous on it, or since the utility converges to $-\infty$ at $q_n = 0$, there exists an optimal solution to each of them.

The Lagrangian of $SYSTEM$ is:

$$L_{SYS}(q, \lambda, \nu) := -\sum_{i=1}^N U_i(q_i) + \sum_{S\subseteq\{1,2,\ldots,N\}} \lambda_S [\sum_{i\in S} \frac{q_i}{p_i} - (\tau - I_S)] - \sum_{i=1}^N \nu_i q_i,$$

where $\lambda := [\lambda_S : S \subseteq \{1, 2, \ldots, N\}]$ and $\nu := [\nu_n : 1 \le n \le N]$ are the Lagrange multipliers. By the KKT condition, a vector $q^* := [q_1^*, q_2^*, \ldots, q_N^*]$ is the optimal solution to SYSTEM if q^* is feasible and there exists vectors λ^* and ν^* such that:

$$\left.\frac{\partial L_{SYS}}{\partial q_n}\right|_{q^*,\lambda^*,\nu^*} = -U_n'(q_n^*) + \frac{\sum_{S\ni n} \lambda_S^*}{p_n} - \nu_n^* = 0, \forall 1 \le n \le N, \tag{9}$$

$$\lambda_S^* [\sum_{i\in S} \frac{q_i^*}{p_i} - (\tau - I_S)] = 0, \forall S \subseteq \{1, 2, \ldots, N\}, \tag{10}$$
$$\nu_n^* q_n^* = 0, \forall 1 \le n \le N, \tag{11}$$
$$\lambda_S^* \ge 0, \forall S \subseteq \{1, \ldots, N\}, \text{ and } \nu_n^* \ge 0, \forall 1 \le n \le N. \tag{12}$$

The Lagrangian of $CLIENT_n$ is:

$$L_{CLI}(\rho_n, \xi_n) := -U_n(\frac{\rho_n}{\psi_n}) + \rho_n - \xi_n \rho_n,$$

where ξ_n is the Lagrange multiplier for $CLIENT_n$. By the KKT condition, ρ_n^* is the optimal solution to $CLIENT_n$ if

$\rho_n^* \geq 0$ and there exists ξ_n^* such that:

$$\frac{dL_{CLI}}{d\rho_n}\bigg|_{\rho_n^*,\xi_n^*} = -\frac{1}{\psi_n}U_n'(\frac{\rho_n^*}{\psi_n}) + 1 - \xi_n^* = 0, \tag{13}$$
$$\xi_n^*\rho_n^* = 0, \tag{14}$$
$$\xi_n^* \geq 0. \tag{15}$$

Finally, the Lagrangian of $ACCESS\text{-}POINT$ is:

$$L_{NET}(q,\zeta,\mu) := -\sum_{i=1}^N \rho_i \log q_i + \sum_{S\subseteq\{1,2,\dots,N\}} \zeta_S[\sum_{i\in S}\frac{q_i}{p_i} - (\tau - I_S)] - \sum_{i=1}^N \mu_i q_i,$$

where $\zeta := [\zeta_S : S \subseteq \{1,2,\dots,N\}]$ and $\mu := [\mu_n : 1 \leq n \leq N]$ are the Lagrange multipliers. Again, by the KKT condition, a vector $q^* := [q_n^*]$ is the optimal solution to $ACCESS\text{-}POINT$ if q^* is feasible and there exists vectors ζ^* and μ^* such that:

$$\frac{\partial L_{NET}}{\partial q_n}\bigg|_{q^*,\zeta^*,\mu^*} = -\frac{\rho_n}{q_n^*} + \frac{\sum_{S\ni n}\zeta_S^*}{p_n} - \mu_n^* = 0, \forall 1 \leq n \leq N, \tag{16}$$
$$\zeta_S^*[\sum_{i\in S}\frac{q_i^*}{p_i} - (\tau - I_S)] = 0, \forall S \subseteq \{1,2,\dots,N\}, \tag{17}$$
$$\mu_n^* q_n^* = 0, \forall 1 \leq n \leq N, \tag{18}$$
$$\zeta_S^* \geq 0, \forall S \subseteq \{1,\dots,N\}, \text{ and } \mu_n^* \geq 0, \forall 1 \leq n \leq N. \tag{19}$$

Let q^* be a solution to $SYSTEM$, and let λ^*, ν^* be the corresponding Lagrange multipliers that satisfy conditions (9)–(12). Let $q_n = q_n^*$, $\rho_n = \frac{\sum_{S\ni n}\lambda_S^*}{p_n}q_n^*$, and $\psi_n = \frac{\sum_{S\ni n}\lambda_S^*}{p_n}$, for all n. Clearly, q, ρ, and ψ are all non-negative vectors. We will show (q,ρ,ψ) satisfy (i) and (ii).

We first show (i) for all n such that $\psi_n = \frac{\sum_{S\ni n}\lambda_S^*}{p_n} > 0$. It is obvious that $\rho_n = \psi_n q_n$. Also, $\rho_n \geq 0$, since $\lambda_S^* \geq 0$ (by (12)) and $q_n^* \geq 0$ (since q^* is feasible). Further, let the Lagrange multiplier of $CLIENT_n$, ξ_n, be equal to $\nu_n^*/\frac{\sum_{S\ni n}\lambda_S^*}{p_n} = \nu_n^*/\psi_n$. Then we have:

$$\begin{aligned}\frac{\partial L_{CLI}}{\partial \rho_n}\bigg|_{\rho_n,\xi_n} &= -\frac{1}{\psi_n}U_n'(\frac{\rho_n}{\psi_n}) + 1 - \xi_n \\ &= \frac{1}{\psi_n}(-U_n'(\frac{\rho_n}{\psi_n}) + \psi_n - \psi_n\xi_n) \\ &= \frac{1}{\psi_n}(-U_n'(q_n^*) + \frac{\sum_{S\ni n}\lambda_S^*}{p_n} - \nu_n^*) = 0, \text{ by (9)},\end{aligned}$$
$$\xi_n\rho_n = \frac{\nu_n^*}{\psi_n}\psi_n q_n^* = \nu_n^* q_n^* = 0, \text{ by (11)}$$
$$\xi_n = \nu_n^*/\frac{\sum_{S\ni n}\lambda_S^*}{p_n} \geq 0, \text{ by (12)}.$$

In sum, (ρ,ψ,ξ) satisfies the KKT conditions for $CLIENT_n$, and therefore ρ_n is a solution to $CLIENT_n$, with $\rho_n = \psi_n q_n$.

Next we establish (ii). Since $q = q^*$ is the solution to $SYSTEM$, it is feasible. Let the Lagrange multipliers of $ACCESS\text{-}POINT$ be $\zeta_S = \lambda_S^*, \forall S$, and $\mu_n = 0, \forall n$, respectively. Given that each client n pays ρ_n per period, we have:

$$\begin{aligned}\frac{\partial L_{NET}}{\partial q_n}\bigg|_{q,\zeta,\mu} &= -\frac{\rho_n}{q_n} + \frac{\sum_{S\ni n}\zeta_S}{p_n} - \mu_n \\ &= -\psi_n + \psi_n - 0 = 0, \forall n,\\ \zeta_S[\textstyle\sum_{i\in S}\frac{q_i}{p_i} - (\tau - I_S)] &= \lambda_S^*[\textstyle\sum_{i\in S}\frac{q_i^*}{p_i} - (\tau - I_S)] \\ &= 0, \forall S, \text{ by (10)},\end{aligned}$$
$$\mu_n q_n = 0 \times q_n = 0, \forall n,$$
$$\zeta_S = \lambda_S^* \geq 0, \forall S \text{ (by (12)), and } \mu_n \geq 0, \forall n.$$

Therefore, (q,ζ,μ) satisfies the KKT condition for $ACCESS\text{-}POINT$ and thus q is a solution to $ACCESS\text{-}POINT$.

For the converse, suppose (q,ρ,ψ) are positive vectors with $\rho_n = \psi_n q_n$, for all n, that satisfy (i) and (ii). We wish to show that q is a solution to $SYSTEM$. Let ξ_n be the Lagrange multiplier for $CLIENT_n$. Since we assume $\psi_n > 0$ for all n, the problem $CLIENT_n$ is well-defined for all n, and so is ξ_n. Also, let ζ and μ be the Lagrange multipliers for $ACCESS\text{-}POINT$. Since $q_n > 0$ for all n, we have $\mu_n = 0$ for all n by (18). By (16), we also have:

$$\begin{aligned}\frac{\partial L_{NET}}{\partial q_n}\bigg|_{q,\zeta,\mu} &= -\frac{\rho_n}{q_n} + \frac{\sum_{S\ni n}\zeta_S}{p_n} - \mu_n \\ &= -\psi_n + \frac{\sum_{S\ni n}\zeta_S}{p_n} = 0,\end{aligned}$$

and thus $\psi_n = \frac{\sum_{S\ni n}\zeta_S}{p_n}$. Let $\lambda_S = \zeta_S$, for all S, and $\nu_n = \psi_n\xi_n$, for all n. We claim that q is the optimal solution to $SYSTEM$ with Lagrange multipliers λ and ν.

Since q is a solution to $ACCESS\text{-}POINT$, it is feasible. Further, we have:

$$\begin{aligned}\frac{\partial L_{SYS}}{\partial q_n}\bigg|_{q,\lambda,\nu} &= -U_n'(q_n) + \frac{\sum_{S\ni n}\lambda_S}{p_n} - \nu_n \\ &= -U_n'(\frac{\rho_n}{\psi_n}) + \psi_n - \psi_n\xi_n = 0, \forall n, \text{ by (13)},\\ \lambda_S[\textstyle\sum_{n\in S}\frac{q_n}{p_n} - (\tau - I_S)] &= \zeta_S[\textstyle\sum_{n\in S}\frac{q_n}{p_n} - (\tau - I_S)] \\ &= 0, \forall S, \text{ by (17)},\end{aligned}$$
$$\nu_n q_n = \xi_n\rho_n = 0, \forall n, \text{ by (14)},$$
$$\lambda_S = \zeta_S \geq 0, \forall S, \text{ by (19)},$$
$$\nu_n = \psi_n\xi_n \geq 0, \forall n, \text{ by (15)}.$$

Thus, (q,λ,ν) satisfy the KKT condition for $SYSTEM$, and so q is a solution to $SYSTEM$. ∎

V. A Bidding Game between Clients and Access Point

Theorem 2 states that the maximum total utility of the system can be achieved when the solutions to the problems $CLIENT_n$ and $ACCESS\text{-}POINT$ agree. In this section, we formulate a repeated game for such reconciliation. We also discuss the meanings of the problems $CLIENT_n$ and $ACCESS\text{-}POINT$ in this repeated game.

The repeated game is formulated as follows:

Step 1: Each client n announces an amount ρ_n that it pays per period.

Step 2: After noting the amounts, $\rho_1, \rho_2, \dots, \rho_N$, paid by the clients, the AP chooses a scheduling policy so that the resulting long-term timely throughput, q_n, for each client maximizes $\sum_{i=1}^{N} \rho_i \log q_i$.

Step 3: The client n observes its own timely throughput, q_n. It computes $\psi_n := \rho_n/q_n$. It then determines $\rho_n^* \geq 0$ to maximize $U_n(\frac{\rho_n^*}{\psi_n}) - \rho_n^*$. Client n updates the amount it pays to $(1-\alpha)\rho_n + \alpha\rho_n^*$, with some fixed $0 < \alpha < 1$, and announces the new bid value.

Step 4: Go back to Step 2.

In Step 3 of the game, client n chooses its new amount of payment as a weighted average of the past amount and the derived optimal value, instead of the derived optimal value. This design serves two purposes. First, it seeks to avoid the system from oscillating between two extreme values. Second, since ρ_n is initiated to a positive value, and ρ_n^* derived in each iteration is always non-negative, this design guarantees ρ_n to be positive throughout all iterations. Since $\psi_n = \rho_n/q_n$, this also ensures $\psi_n > 0$ and the function $U_n(\frac{\rho_n}{\psi_n})$ is consequently always well-defined.

We show that the fixed point of this repeated game maximizes the total utility of the system:

Theorem 3: Suppose at the fixed point of the repeated game, each client n pays ρ_n^* per period, and receives timely throughput q_n^*. If both ρ_n^* and q_n^* are positive for all n, the vector q^* maximizes the total utility of the system.

Proof: Let $\psi_n^* = \frac{\rho_n^*}{q_n^*}$. It is positive since both ρ_n^* and q_n^* are positive. Since the vectors q^* and ρ^* are derived from the fixed point, ρ_n^* maximizes $U_n(\frac{\rho_n}{\psi_n^*}) - \rho_n$, over all $\rho_n \geq 0$, as described in Step 3 of the game. Thus, ρ_n^* is a solution to $CLIENT_n$, given $\rho_n^* = \psi_n^* q_n^*$. Similarly, from Step 2, q^* is the feasible vector that maximizes $\sum_{i=1}^{N} \rho_i^* \log q_i$, over all feasible vectors q. Thus, q^* is a solution to $ACCESS$-$POINT$, given that each client n pays ρ_n^* per period. By Theorem 2, q^* is the unique solution to $SYSTEM$ and therefore maximizes the total utility of the system. ■

Next, we describe the meaning of the game. In Step 3, client n assumes a linear relation between the amount it pays, ρ_n, and the timely throughput it receives, q_n. To be more exactly, it assumes $\rho_n = \psi_n q_n$, where ψ_n is the price. Thus, maximizing $U_n(\frac{\rho_n}{\psi_n}) - \rho_n$ is equivalent to maximizing $U_n(q_n) - \rho_n$. Recall that $U_n(q_n)$ is the utility that client n obtains when it receives timely throughput q_n. $U_n(q_n) - \rho_n$ is therefore the net profit that client n gets. In short, in Step 3, the goal of client n is to selfishly maximize its own net profit using a first order linear approximation to the relation between payment and timely throughput.

We next discuss the behavior of the AP in Step 2. The AP schedules clients so that the resulting timely throughput vector q is a solution to the problem $ACCESS$-$POINT$, given that each client n pays ρ_n per period. Thus, q is feasible and there exists vectors ζ and μ that satisfy conditions (16)–(19). While it is difficult to solve this problem, we consider a special restrictive case that gives us a simple solution and insights into the AP's behavior. Let $TOT := \{1, 2, \ldots, N\}$ be the set that consists of all clients. We assume that a solution (q, ζ, μ) to the problem has the following properties: $\zeta_S = 0$, for all $S \neq TOT$, $\zeta_{TOT} > 0$, and $\mu_n = 0$, for all n. By (16), we have:

$$-\frac{\rho_n}{q_n} + \frac{\sum_{S \ni n} \zeta_S}{p_n} - \mu_n = -\frac{\rho_n}{q_n} + \frac{\zeta_{TOT}}{p_n} = 0,$$

and therefore $q_n = p_n \rho_n / \zeta_{TOT}$. Further, since $\zeta_{TOT} > 0$, (17) requires that:

$$\sum_{i \in TOT} \frac{q_i}{p_i} - (\tau - I_{TOT}) = \sum_{i \in TOT} \frac{\rho_i}{\zeta_{TOT}} - (\tau - I_{TOT}) = 0.$$

Thus, $\zeta_{TOT} = \frac{\sum_{i=1}^{N} \rho_i}{\tau - I_{TOT}}$ and $\frac{q_n}{p_n} = \frac{\rho_n}{\sum_{i=1}^{N} m_i}(\tau - I_{TOT})$, for all n. Notice that the derived (q, ζ, μ) satisfies conditions (16)–(19). Thus, under the assumption that q is feasible, this special case actually maximizes $\sum_{i=1}^{N} \rho_i \log q_i$. In Section VI we will address the general situation without any such assumption, since it needs not be true.

Recall that I_{TOT} is the average number of time slots that the AP is forced to be idle in a period after it has completed all clients. Also, by Lemma 1, $\frac{q_n}{p_n}$ is the workload of client n, that is, the average number of time slots that the AP should spend working for client n. Thus, letting $\frac{q_n}{p_n} = \frac{\rho_n}{\sum_{i=1}^{N} \rho_i}(\tau - I_{TOT})$, for all n, the AP tries to allocate those non-idle time slots so that the average number of time slots each client gets is proportional to its payment. Although we only study the special case of I_{TOT} here, we will show that the same behavior also holds for the general case in the Section VI.

In summary, the game proposed in this section actually describes a bidding game, where clients are bidding for non-idle time slots. Each client gets a share of time slots that is proportional to its bid. The AP thus assigns timely throughputs, based on which the clients calculate a price and selfishly maximize their own net profits. Finally, Theorem 3 states that the equilibrium point of this game maximizes the total utility of the system.

VI. A Scheduling Policy for Solving $ACCESS$-$POINT$

In Section V, we have shown that by setting $q_n = p_n \frac{\rho_n}{\sum_{i=1}^{N} m_i}(\tau - I_{TOT})$, the resulting vector q solves $ACCESS$-$POINT$ provided q is indeed feasible. Unfortunately, such q is not always feasible and solving $ACCESS$-$POINT$ is in general difficult. Even for the special case discussed in Section V, solving $ACCESS$-$POINT$ requires knowledge of channel conditions, that is, p_n. In this section, we propose a very simple priority based scheduling policy that can achieve the optimal solution for $ACCESS$-$POINT$, and that too without any knowledge of the channel conditions.

In the special case discussed in Section V, the AP tries, though it may be impossible in general, to allocate non-idle time slots to clients in proportion to their payments. Based on this intuitive guideline, we design the following scheduling policy. Let $u_n(t)$ be the number of time slots that the AP has allocated for client n up to time t. At the beginning of each period, the AP sorts all clients in increasing order of $\frac{u_n(t)}{\rho_n}$, so that $\frac{u_1(t)}{\rho_1} \leq \frac{u_2(t)}{\rho_2} \leq \ldots$ after renumbering clients if necessary. The AP then schedules transmissions according to the priority ordering, where clients with smaller $\frac{u_n(t)}{\rho_n}$ get higher priorities. Specifically, in each time slot during the period, the AP chooses the smallest i for which the packet for client i is not yet delivered, and then transmits the packet for client i in that time slot. We call this the *weighted transmission* policy (WT). Notice that the policy only requires the AP to keep track of the bids of clients and the number of time slots each client has been allocated in the past, followed by a sorting of $\frac{u_n(t)}{\rho_n}$ among all clients. Thus, the policy requires no information on the actual channel conditions, and is tractable. Simple as it is, we show that the policy actually achieves the optimal solution for $ACCESS\text{-}POINT$. In the following sections, we first prove that the vector of timely throughputs resulting from the WT policy converges to a single point. We then prove that this limit is the optimal solution for $ACCESS\text{-}POINT$. Finally, we establish that the WT policy additionally achieves some forms of fairness.

A. Convergence of the Weighted Transmission Policy

We now prove that, by applying the WT policy, the timely throughputs of clients will converge to a vector q. To do so, we actually prove the convergence property and precise limit of a more general class of scheduling policies, which not only consists of the WT policy but also a scheduling policy proposed in [8]. The proof is similar to that used in [8] and is based on Blackwell's approachability theorem [3]. The proof in [8] only shows that the vector of timely throughputs approaches a desirable set in the N-space under a particular policy, while here we prove that the vector of timely throughputs converges to a single point under a more general class of scheduling policies. Thus, our result is both stronger and more general than the one in [8].

We start by introducing Blackwell's approachability theorem. Consider a single player repeated game. In each round i of the game, the player chooses some action, $a(i)$, and receives a reward $v(i)$, which is a random vector whose distribution is a function of $a(i)$. Blackwell studies the long-term average of the rewards received, $\lim_{j\to\infty}\sum_{i=1}^{j} v(i)/j$, defining a set as *approachable*, under some policy, if the distance between $\sum_{i=1}^{j} v(i)/j$ and the set converges to 0 with probability one, as $j\to\infty$.

Theorem 4 (Blackwell [3]): Let $A\subseteq\mathbb{R}^N$ be any closed set. Suppose that for every $x\notin A$, a policy η chooses an action a $(=a(x))$, which results in an expected payoff vector $E(v)$. If the hyperplane through y, the closest point in A to x, perpendicular to the line segment xy, separates x from $E(v)$, then A is approachable with the policy η.

Now we formulate our more general class of scheduling policies. We call a policy a *generalized transmission time policy* if, for a choice of a positive parameter vector a and non-negative parameter vector b, the AP sorts clients by $a_n u_n(t)-b_n t$ at the beginning of each period, and gives priorities to clients with lower values of this quantity. Note that the special case $a_n\equiv\frac{1}{\rho_n}$ and $b_n\equiv 0$ yields the WT policy, while the choice $a_n\equiv 1$ and $b_n\equiv\frac{q_n}{p_n}$ yields the largest time-based debt first policy of [8], and thus we describe a more general set of policies.

Theorem 5: For each generalized transmission time policy, there exists a vector q such that the vector of work loads resulting from the policy converges to $w(q):=[w_n(q_n)]$.

Proof: Given the parameters $\{(a_n,b_n):1\le n\le N\}$, we give an exact expression for the limiting q. We define a sequence of sets $\{H_k\}$ and corresponding values $\{\theta_k\}$ iteratively as follows. Let $H_0:=\phi$, $\theta_0:=-\infty$, and

$$H_k:=\arg\min_{S:S\supsetneqq H_{k-1}}\frac{\frac{1}{\tau}(I_{H_{k-1}}-I_S)-\sum_{n\in S\setminus H_{k-1}}\frac{b_n}{a_n}}{\sum_{n\in S\setminus H_{k-1}}1/a_n},$$

$$\theta_k:=\frac{\frac{1}{\tau}(I_{H_{k-1}}-I_{H_k})-\sum_{n\in H_k\setminus H_{k-1}}\frac{b_n}{a_n}}{\sum_{n\in H_k\setminus H_{k-1}}1/a_n},\ \text{for all } k>0.$$

In selecting H_k, we always choose a maximal subset, breaking ties arbitrary. $(H_1,\theta_1),(H_2,\theta_2),\dots,$ can be iteratively defined until every client is in some H_k. Also, by the definition, we have $\theta_k>\theta_{k-1}$, for all $k>0$. If client n is in $H_k\setminus H_{k-1}$, we define $q_n:=\tau p_n\frac{b_n+\theta_k}{a_n}$, and so $w_n(q_n)=\tau\frac{b_n+\theta_k}{a_n}$. The proof of convergence consists of two parts. First we prove that the vector of work performed (see Lemma 1 for definition) approaches the set $\{w^*|w^*_n\ge w_n(q_n)\}$. Then we prove that $w(q)$ is the only feasible vector in the set $\{w^*|w^*_n\ge w_n(q_n)\}$. Since the *feasible region* for work loads, defined as the set of all feasible vectors for work loads, is approachable under any policy, the vector of work performed resulting from the generalized transmission time policy must converge to $w(q)$.

For the first part, we prove the following statement: for each $k\ge 1$, the set $W_k:=\{w^*|w^*_n\ge\tau\frac{b_n+\theta_k}{a_n},\forall n\notin H_{k-1}\}$ is approachable. Since $\cap_{i\ge 0}W_i=\{w^*|w^*_n\ge w_n(q_n)\}$, we also prove that $\{w^*|w^*_n\ge w_n(q_n)\}$ is approachable.

Consider a linear transformation on the space of workloads $L(w):=[l_n:l_n=\frac{a_n w_n/\tau-b_n}{\sqrt{a_n}}]$. Proving W_k is approachable is equivalent to proving that its image under L, $V_k:=\{l|l_n\ge\frac{\theta_k}{\sqrt{a_n}},\forall n\notin H_{k-1}\}$, is approachable. Now we apply Blackwell's theorem. Suppose at some time t that is the beginning of a period, the number of time slots that the AP has worked on client n is $u_n(t)$. The work performed for client n is $\frac{u_n(t)}{t/\tau}$, and the image of the vector of work performed under L is $x(t):=[x_n(t)|x_n(t)=\frac{a_n u_n(t)/t-b_n}{\sqrt{a_n}}]$, which we shall suppose is not in V_k. The generalized transmission time policy sorts clients so that $a_1u_1(t)-b_1\le a_2u_2(t)-b_2\le\dots$, or equivalently, $\sqrt{a_1}x_1(t)\le\sqrt{a_2}x_2(t)\le\dots$. The closest point in V_k to $x(t)$ is $y:=[y_n]$, where $y_n=\frac{\theta_k}{\sqrt{a_n}}$, if $x_n(t)<\frac{\theta_k}{\sqrt{a_n}}$ and $n\notin H_{k-1}$, and $y_n=x_n$, otherwise. The hyperplane that passes through y and is orthogonal to the line segment xy is:

$$\{z|f(z):=\sum_{n:n\le n_0,n\notin H_{k-1}}(z_n-\frac{\theta_k}{\sqrt{a_n}})(x_n(t)-\frac{\theta_k}{\sqrt{a_n}})=0\}.$$

Let π_n be the expected number of time slots that the AP spends on working for client n in this period under the generalized transmission time policy. The image under L of the expected reward in this period is $\pi_L:=[\frac{a_n\pi_n/\tau-b_n}{\sqrt{a_n}}]$.

Blackwell's theorem shows that V_k is approachable if $x(t)$ and π_L are separated by the plane $\{z|f(z)=0\}$. Since $f(x(t))\geq 0$, it suffices to show $f(\pi_L)\leq 0$.

We manipulate the original ordering, for this period, so that all clients in H_{k-1} have higher priorities than those not in H_{k-1}, while preserving the relative ordering between clients not in H_{k-1}. Note this manipulation will not give any client $n\notin H_{k-1}$ higher priority than it had in the original ordering. Therefore, π_n will not increase for any $n\notin H_{k-1}$. Since the value of $f(\pi_L)$ only depends on π_n for $n\notin H_{k-1}$, and increases as those π_n decrease, this manipulation will not decrease the value of $f(\pi_L)$. Thus, it suffices to prove that $f(\pi_L)\leq 0$, under this new ordering. Let $n_0:=|H_{k-1}|+1$. Under this new ordering, we have: $\sqrt{a_{n_0}}x_{n_0}(t)\leq\sqrt{a_{n_0+1}}x_{n_0+1}(t)\leq\cdots\leq\sqrt{a_{n_1}}x_{n_1}(t)<\theta_k\leq\sqrt{a_{n_1+1}}x_{n_1+1}(t)\leq\dots$.

Let $\delta_n=\sqrt{a_n}x_n(t)-\sqrt{a_{n+1}}x_{n+1}(t)$, for $n_0\leq n\leq n_1-1$ and $\delta_{n_1}=\sqrt{a_{n_1}}x_{n_1}(t)-\theta_k$. Clearly, $\delta_n\leq 0$, for all $n_0\leq n\leq n_1$. Now we can derive:

$$\begin{aligned}f(\pi_L)&=\sum_{n=n_0}^{n_1}\Big(\frac{a_n\pi_n/\tau-b_n}{\sqrt{a_n}}-\frac{\theta_k}{\sqrt{a_n}}\Big)\Big(x_n(t)-\frac{\theta_k}{\sqrt{a_n}}\Big)\\&=\sum_{n=n_0}^{n_1}\Big(\frac{\pi_n}{\tau}-\frac{b_n}{a_n}-\frac{\theta_k}{a_n}\Big)(\sqrt{a_n}x_n(t)-\theta_k)\\&=\sum_{i=n_0}^{n_1}\Big(\frac{\sum_{n=n_0}^{i}\pi_n}{\tau}-\sum_{n=n_0}^{i}\frac{b_n}{a_n}-\theta_k\sum_{n=n_0}^{i}\frac{1}{a_n}\Big)\delta_i.\end{aligned}$$

Recall that I_S is the expected number of idle time slots when the AP only caters on the subset S. Thus, under this ordering, we have $\sum_{n=1}^{i}\pi_n=\tau-I_{\{1,\dots,i\}}$, for all i, and $\sum_{n=n_0}^{i}\pi_n=I_{\{1,\dots,n_0-1\}}-I_{\{1,\dots,i\}}=I_{H_{k-1}}-I_{\{1,\dots,i\}}$, for all $i\geq n_0$. By the definition of H_k and θ_k, we also have

$$\begin{aligned}&\frac{\sum_{n=n_0}^{i}\pi_n}{\tau}-\sum_{n=n_0}^{i}\frac{b_n}{a_n}-\theta_k\sum_{n=n_0}^{i}\frac{1}{a_n}\\=&\Big(\sum_{n=n_0}^{i}\frac{1}{a_n}\Big)\Big(\frac{\frac{1}{\tau}(I_{H_{k-1}}-I_{\{1,\dots,i\}})-\sum_{n=n_0}^{i}\frac{b_n}{a_n}}{\sum_{n\in\{1,\dots,i\}\backslash H_{k-1}}1/a_n}-\theta_k\Big)\geq 0.\end{aligned}$$

Therefore, $f(\pi_L)\leq 0$, since $\delta_i\leq 0$, and V_k is indeed approachable, for all k.

We have established that the set $\{w^*|w^*_n\geq w_n(q_n)\}$ is approachable. Next we prove that $[w_n(q_n)]$ is the only feasible vector in the set. Consider any vector $w'\neq w(q)$ in the set. We have $w'_n\geq w_n(q_n)$ for all n, and $w'_{n_0}>w_{n_0}(q_{n_0})$, for some n_0. Suppose $n_0\in H_k\backslash H_{k-1}$. We have:

$$\begin{aligned}\sum_{n\in H_k}w'_n>\sum_{n\in H_k}w_n(q_n)&=\sum_{i=1}^{k}\sum_{n\in H_i\backslash H_{i-1}}\tau\frac{b_n+\theta_k}{a_n}\\&=\sum_{i=1}^{k}(I_{H_{i-1}}-I_{H_i})=\tau-I_{H_k},\end{aligned}$$

and thus w' is not feasible. Therefore, $w(q)$ is the only feasible vector in $\{w^*|w^*_n\geq w_n(q_n)\}$, and the vector of work performed resulting from the generalized transmission time policy must converge to $w(q)$. ∎

Corollary 1: For the policy of Theorem 5, the vector of timely throughputs converges to q.

Proof: Follows from Lemma 1. ∎

B. Optimality of the Weighted Transmission Policy for $ACCESS\text{-}POINT$

Theorem 6: Given $[\rho_n]$, the vector q of long-term average timely throughputs resulting from the WT policy is a solution to $ACCESS\text{-}POINT$.

Proof: We use the sequence of sets $\{H_k\}$ and values $\{\theta_k\}$, with $a_n:=\frac{1}{\rho_n}$ and $b_n:=0$, as defined in the proof of Theorem 5. Let $K:=|\{\theta_k\}|$. Thus, we have $H_K=TOT=\{1,2,\dots,N\}$. Also, let $m_k:=|H_k|$. For convenience, we renumber clients so that $H_k=\{1,2,\dots,m_k\}$. The proof of Theorem 5 shows that $q_n=\tau p_n\theta_k\rho_n$, for $n\in H_k\backslash H_{k-1}$. Therefore, $w_n(q_n)=\frac{q_n}{p_n}=\tau\theta_k\rho_n$. Obviously, q is feasible, since it is indeed achieved by the WT policy. Thus, to establish optimality, we only need to prove the existence of vectors ζ and μ that satisfy conditions (16)–(19).

Set $\mu_n=0$, for all n. Let $\zeta_{H_K}=\zeta_{TOT}:=\frac{\rho_N}{w_N(q_N)}=\frac{1}{\tau\theta_K}$ and $\zeta_{H_k}:=\frac{\rho_{m_k}}{w_{m_k}(q_{m_k})}-\frac{\rho_{m_{k+1}}}{w_{m_{k+1}}(q_{m_{k+1}})}=\frac{1}{\tau\theta_k}-\frac{1}{\tau\theta_{k+1}}$, for $1\leq k\leq K-1$. Finally, let $\zeta_S:=0$, for all $S\notin\{H_1,H_2,\dots,H_K\}$. We claim that the vectors ζ and μ, along with q, satisfy conditions (16)–(19).

We first evaluate condition (16). Suppose client n is in $H_k\backslash H_{k-1}$. Then client n is also in $H_{k+1},H_{k+2},\dots,H_K$. So,

$$\begin{aligned}&-\frac{\rho_n}{q_n}+\frac{\sum_{S\ni n}\zeta_S}{p_n}-\mu_n=-\frac{1}{\tau\theta_k p_n}+\frac{\sum_{i=k}^{K}\zeta_{H_i}}{p_n}\\&=-\frac{1}{\tau\theta_k p_n}+\frac{1}{\tau\theta_k p_n}=0.\end{aligned}$$

Thus, condition (16) is satisfied.

Since $\mu_n=0$, for all n, condition (18) is satisfied. Further, since $\frac{1}{\theta_k}>\frac{1}{\theta_{k+1}}$, for all $1\leq k\leq K-1$, condition (19) is also satisfied. It remains to establish condition (17). Since $\zeta_S=0$ for all $S\notin\{H_1,H_2,\dots,H_K\}$, we only need to show $\sum_{i\in S}\frac{q_i}{p_i}-(\tau-I_S)=0$ for $S\in\{H_1,H_2,\dots,H_K\}$.

Consider H_k. For each client $i\in H_k$ and each client $j\notin H_k$, $\frac{w_i(q_i)}{\rho_i}<\frac{w_j(q_j)}{\rho_j}$. Since $w_n(q_n)$ is the average number of time slots that the AP spends on working for client n, we have $\frac{u_i(t)}{\rho_i}<\frac{u_j(t)}{\rho_j}$, for all $i\in H_k$ and $j\notin H_k$, after a finite number of periods. Therefore, except for a finite number of periods, clients in H_k will have priorities over those not in H_k. In other words, if we only consider the behavior of those clients in H_k, it is the same as if the AP only works on the subset H_k of clients. Further, recall that I_{H_k} is the expected number of time slots that the AP is forced to stay idle when the AP only works on the subset H_k of clients. Thus, we have $\sum_{i\in H_k}w_i(q_i)=\tau-I_{H_k}$ and $\sum_{i\in H_k}\frac{q_i}{p_i}-(\tau-I_{H_k})=0$, for all k. ∎

C. Fairness of Allocated Timely Throughputs

We now show that the WT policy not only solves the $ACCESS\text{-}POINT$ problem but also achieves some forms of fairness among clients. Two common fairness criteria are *max-min fair* and *proportionally fair*. We extend the definitions of these two criteria as follows:

Definition 3: A scheduling policy is called *weighted max-min fair with positive weight vector* $a=[a_n]$ if it achieves q, and, for any other feasible vector q', we have: $q'_i>q_i\Rightarrow q'_j<q_j$, for some j such that $\frac{w_i(q_i)}{a_i}\geq\frac{w_j(q_j)}{a_j}$.

Definition 4: A scheduling policy is called *weighted proportionally fair with positive weight vector* a if it achieves q and, for any other feasible vector q', we have:

$$\sum_{n=1}^{N} \frac{w_n(q'_n) - w_n(q_n)}{w_n(q_n)/a_n} \le 0.$$

Next, we prove that the WT policy is both weighted max-min fair and proportionally fair with weight vector ρ.

Theorem 7: The weighted transmission policy is weighted max-min fair with weight ρ

Proof: We sort clients and define $\{H_k\}$ as in the proof of Theorem 6. Let q be the vector achieved by the WT policy and q' be any feasible vector. Suppose $q'_i > q_i$ for some i. Assume client i is in $H_k \backslash H_{k-1}$. The proof in Theorem 6 shows that $\sum_{n \in H_k} w_n(q_n) = \tau - I_{H_k}$. On the other hand, the feasibility condition requires $\sum_{n \in H_k} w_n(q'_n) \le \tau - I_{H_k} = \sum_{n \in H_k} w_n(q_n)$. Further, since $q'_i > q_i$, $w_i(q'_i) > w_i(q_i)$, there must exist some $j \in H_k$ so that $w_j(q'_j) < w_j(q_j)$, that is, $q'_j < q_j$. Finally, since $i \in H_k \backslash H_{k-1}$, we have $\frac{w_i(q_i)}{\rho_i} \ge \frac{w_n(q_n)}{\rho_n}$, for all $n \in H_k$, and hence $\frac{w_i(q_i)}{\rho_i} \ge \frac{w_j(q_j)}{\rho_j}$. ■

Theorem 8: The weighted transmission policy is proportionally fair with weight ρ.

Proof: We sort clients and define $\{H_k\}$ as in the proof of Theorem 6. Let q be the vector achieved by the WT policy, and let q' be any feasible vector. We have $\frac{w_i(q_i)}{\rho_i} = \tau\theta_k$, if $i \in H_k \backslash H_{k-1}$. Define $\Delta_k := \sum_{n \in H_k \backslash H_{k-1}} w_n(q'_n) - w_n(q_n)$.

To prove the theorem, we prove a stronger statement by induction:

$$\sum_{n \in H_k} \frac{w_n(q'_n) - w_n(q_n)}{w_n(q_n)/\rho_n} = \sum_{i=1}^{k} \frac{\Delta_i}{\tau\theta_i} \le 0, \text{ for all } k > 0.$$

First consider the case $k = 1$. The proof in Theorem 6 shows that $\sum_{n \in H_1} w_n(q_n) = \tau - I_{H_1}$. Further, the feasibility condition requires $\sum_{n \in H_1} w_n(q'_n) \le \tau - I_{H_1} = \sum_{n \in H_1} w_n(q_n) = \tau - I_{H_1}$, and so $\Delta_1 = \sum_{n \in H_1} w_n(q'_n) - w_n(q_n) \le 0$. Thus, we have $\frac{\Delta_1}{\tau\theta_1} \le 0$.

Suppose we have $\sum_{i=1}^{k} \frac{\Delta_i}{\tau\theta_i} \le 0$, for all $k \le k_0$. Again, the proof in Theorem 6 gives us $\sum_{n \in H_{k_0+1}} w_n(q_n) = \tau - I_{H_{k_0+1}}$ and the feasibility condition requires $\sum_{n \in H_{k_0+1}} w_n(q'_n) \le \tau - I_{H_{k_0+1}} = \sum_{n \in H_{k_0+1}} w_n(q_n)$. Thus, $\sum_{i=1}^{k_0+1} \Delta_i \le 0$. We can further derive:

$$\begin{aligned}
&\sum_{i=1}^{k_0+1} \frac{\Delta_i}{\tau\theta_i} \\
\le &\sum_{i=1}^{k_0} \frac{\Delta_i}{\tau\theta_i}\left(1 - \frac{\theta_i}{\theta_{k_0+1}}\right) \quad \left(\text{since } \sum_{i=1}^{k_0+1} \frac{\Delta_i}{\tau\theta_{k_0+1}} \le 0\right) \\
= &\sum_{j=1}^{k_0} \left[\left(\frac{\theta_{j+1} - \theta_j}{\theta_{k_0+1}}\right) \sum_{i=1}^{j} \frac{\Delta_i}{\tau\theta_i}\right] \\
\le &0 \quad \left(\text{since } \sum_{i=1}^{j} \frac{\Delta_i}{\tau\theta_i} \le 0, \text{ and } \theta_{j+1} > \theta_j, \forall j \le k_0\right)
\end{aligned}$$

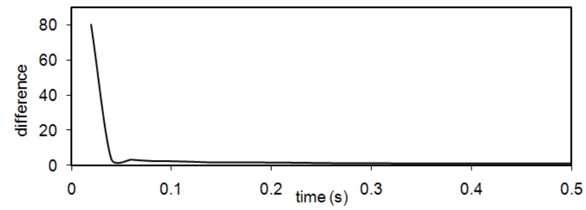


Fig. 1: Convergence of the weighted transmission policy

By induction, $\sum_{i=1}^{k} \frac{\Delta_i}{\tau\theta_i} \le 0$, for all k. Finally, we have:

$$\sum_{n=1}^{N} \frac{w_n(q'_n) - w_n(q_n)}{w_n(q_n)/\rho_n} = \sum_{i=1}^{K} \frac{\Delta_i}{\tau\theta_i} \le 0,$$

and the WT policy is proportionally fair with weight ρ. ■

VII. Simulation Results

We have implemented the WT policy and the bidding game, as described in Section V, on ns-2. We use the G.711 codec for audio compression to set the simulation parameters, as summarized in Table I. All results in this section are averages of 20 simulation runs.

TABLE I: Simulation Setup

Packetization interval	20 ms
Payload size per packet	160 Bytes
Transmission data rate	11 Mb/s
Transmission time (including MAC overheads)	610 μs
# of time slots in a period	32

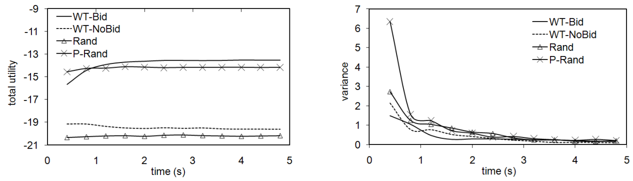
A. Convergence Time for the Weighted Transmission Policy

We have proved that the vector of timely throughputs will converge under the WT policy in Section VI-A. However, the speed of convergence is not discussed. In the bidding game, we assume that the timely throughput observed by each client is post convergence. Thus, it is important to verify whether the WT policy converges quickly. In this simulation, we assume that there are 30 clients in the system. The n^{th} client has channel reliability $(50 + n)\%$ and offers a bid $\rho_n = (n \mod 2) + 1$. We run each simulation for 10 seconds simulation time and then compare the absolute difference of $\sum_n \rho_n \log q_n$ between the timely throughputs at the end of each period with those after 10 seconds. In particular, we artificially set $q_n = 0.001$ if the timely throughput for client n is zero, to avoid computation error for $\log q_n$.

Simulation results are shown in Fig. 1. It can be seen that the timely throughputs converge rather quickly. At time 0.2 seconds, the difference is smaller than 1.4, which is less than 10% of the final value. Based on this observation, we assume that each client updates its bid every 0.2 seconds in the following simulations.

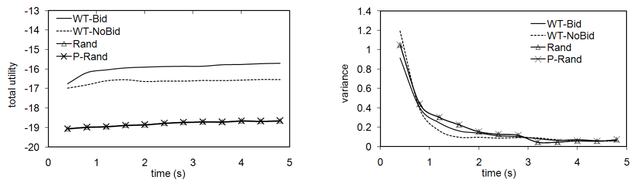
B. Utility Maximization

In this section, we study the total utility that is achieved by iterating between the bidding game and the WT policy, which we call WT-Bid. We assume that the utility function of each client n is given by $\gamma_n \frac{q_n^{\alpha_n} - 1}{\alpha_n}$, where γ_n is a



(a) Average of total utility (b) Variance of total utility

Fig. 2: Performance of the first setting



(a) Average of total utility (b) Variance of total utility

Fig. 3: Performance of the second setting

positive integer and $0 < \alpha_n < 1$. This utility function is strictly increasing, strictly concave, and differentiable for any γ_n and α_n. In addition to evaluating the policy WT-Bid, we also compare the results of three other policies: a policy that employs the WT policy but without updating the bids from clients, which we call WT-NoBid; a policy that decides priorities randomly among clients at the beginning of each period, which we call Rand; and a policy that gives clients with larger γ_n higher priorities, with ties broken randomly, which we call P-Rand.

In each simulation, we assume there are 30 clients. We consider two settings. In the first setting, the n^{th} client has channel reliability $p_n = (50 + n)\%$, $\gamma_n = (n \mod 3) + 1$, and $\alpha_n = 0.3 + 0.1(n \mod 5)$. In the second setting, the n^{th} client has channel reliability $p_n = (20 + 2n)\%$, $\gamma_n = 1$, and $\alpha_n = 0.3 + 0.1(n \mod 5)$. Roughly speaking, the utility functions in the first setting differ much from client to client, while the second setting treats the case when utility functions are similar between clients. In addition to plotting the average of total utility over all simulation runs, we also plot the variance of total utility.

Fig. 2 and Fig. 3 show the simulation results. The WT-Bid policy not only achieves the highest average total utilities but also small variances in both settings. This result suggests that the WT-Bid policy converges very fast. On the other hand, the WT-NoBid policy fails to provide satisfactory performance since it does not consider the different utility functions that clients may have. The P-Rand policy offers good performance in the first setting since it correctly gives higher priority to clients with higher γ_n. Still, it cannot differentiate between clients with the same γ_n and thus results in poor performance in the second setting.

VIII. Concluding Remarks

We have studied the problem of utility maximization problem for clients that demand delay-based QoS support from an access point. Based on an analytical model for QoS support proposed in previous work, we formulate the utility maximization problem as a convex optimization problem. We decompose the problem into two simpler subproblems, namely, $CLIENT_n$ and $ACCESS$-$POINT$. We have proved that the total utility of the system can be maximized by jointly solving the two subproblems. We also describe a bidding game to reconciliate the two subproblems. In the game, each client announces its bid to maximize its own net profit and the AP allocates time slots to achieve the optimal point of $ACCESS$-$POINT$. We have proved that the equilibrium point of the bidding game jointly solves the two subproblems, and therefore achieves the maximum total utility.

In addition, we have proposed a very simple, priority-based weighted transmission policy for solving the $ACCESS$-$POINT$ subproblem. This policy does not require that the AP know the channel reliabilities of the clients, or their individual utilities. We have proved that the long-term performance of a general class of priority-based policies that includes our proposed policy converges to a single point. We then proved that the limiting point of the proposed scheduling policy is the optimal solution to $ACCESS$-$POINT$. Moreover, we have also proved that the resulting allocation by the AP satisfies some forms of fairness criteria. Finally, we have implemented both the bidding game and the scheduling policy in ns-2. Simulation results suggests that the scheduling policy quickly results in convergence. Further, by iterating between the bidding game and the WT policy, the resulting total utility is higher than other tested policies.

References

[1] M.H. Ahmed. Call admission control inwireless networks: A comprehensive survey. *IEEE Communications Surveys*, 7(1):50–69, 2005.

[2] G. Bianchi, A. Campbell, and R. Liao. On utility-fair adaptive services in wireless networks. In *Proc. of IWQoS*, pages 256–267, 1998.

[3] David Blackwell. An analog of the minimax theorem for vector payoffs. *Pacific J. Math*, 6(1), 1956.

[4] Y. Cao and V. Li. Utility-oriented adaptive QoS and bandwidth allocation in wireless networks. In *Proc. of ICC*, 2002.

[5] Y. Cao and V.O.K. Li. Scheduling algorithms in broadband wireless networks. *Proceedings of the IEEE*, 89(1):76–87, Jan. 2001.

[6] H. Fattah and C. Leung. An overview of scheduling algorithms in wireless multimedia networks. *IEEE Wireless Communications*, 9(5):76–83, Oct. 2002.

[7] D. Gao, J. Cai, and K.N. Ngan. Admission control in IEEE 802.11e wireless LANs. *IEEE Network*, pages 6–13, July/August 2005.

[8] I-H. Hou, V. Borkar, and P. R. Kumar. A theory of QoS for wireless. In *Proc. of INFOCOM*, 2009.

[9] I-H. Hou and P.R. Kumar. Admission control and scheduling for QoS guarantees for variable-bit-rate applications on wireless channels. In *Proc. of ACM MobiHoc*, pages 175–184, 2009.

[10] F. Kelly. Charging and rate control for elastic traffic. *European Trans. on Telecommunications*, 8:33–37, 1997.

[11] F.P. Kelly, A.K. Maulloo, and D.K.H. Tan. Rate control in communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, 49:237–252, 1998.

[12] X. Lin and N.B. Shroff. Utility maximization for communication networks with multipath routing. *IEEE Trans. on Automated Control*, 51(5):766–781, 2006.

[13] Q. Ni, L. Romdhani, and T. Turletti. A survey of QoS enhancements for IEEE 802.11 wireless LAN. *Journal of Wireless Communications and Mobile Computing*, 4(5):547–566, 2004.

[14] D. Niyato and E. Hossain. Call admission control for QoS provisioning in 4G wireless networks: issues and approaches. *IEEE Network*, pages 5–11, September/October 2005.

[15] M. Xiao, N.B. Shroff, and E. Chong. A utility-based power-control scheme in wireless cellular systems. *IEEE/ACM Trans. on Networking*, 11(2):210–221, 2003.