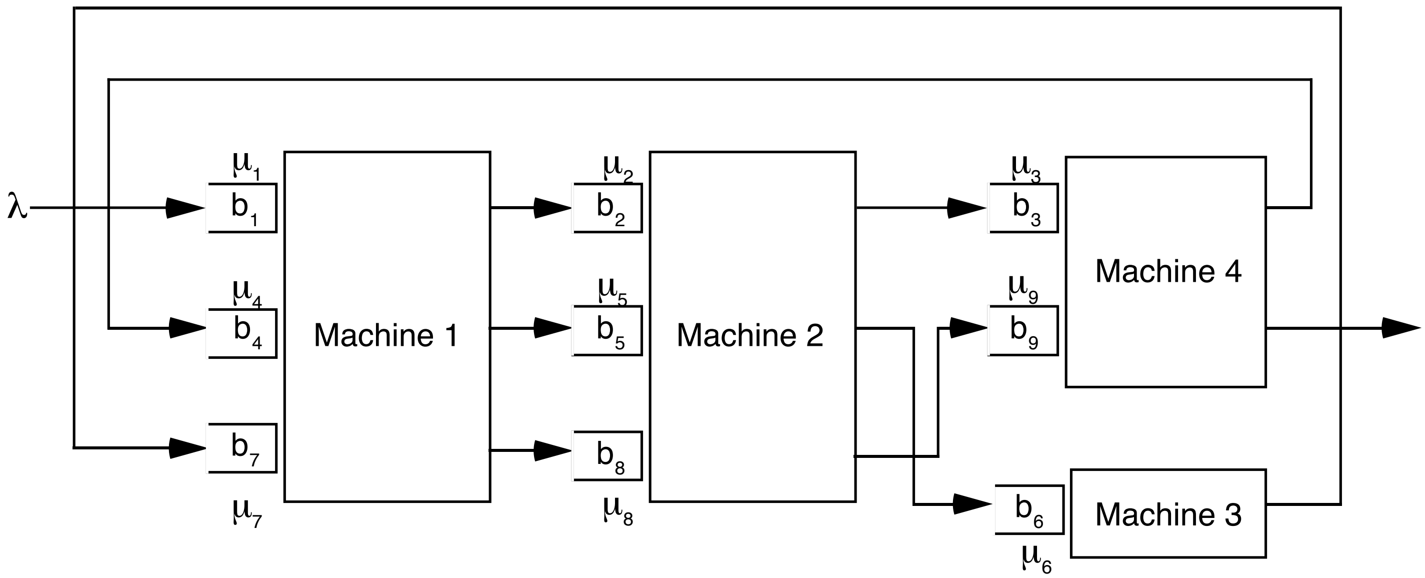

Figure 1: A Basic Open Re-Entrant Line

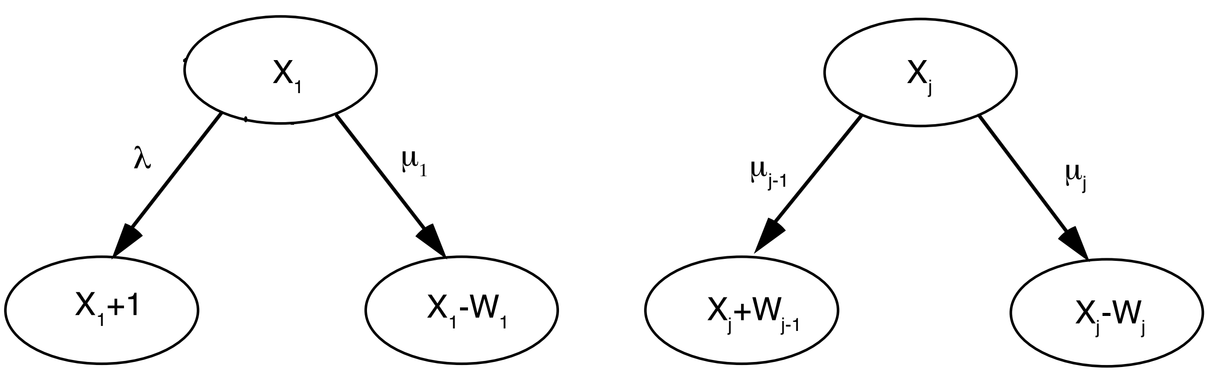

Figure 2: State Transition Diagram for Basic Open Re-Entrant Line

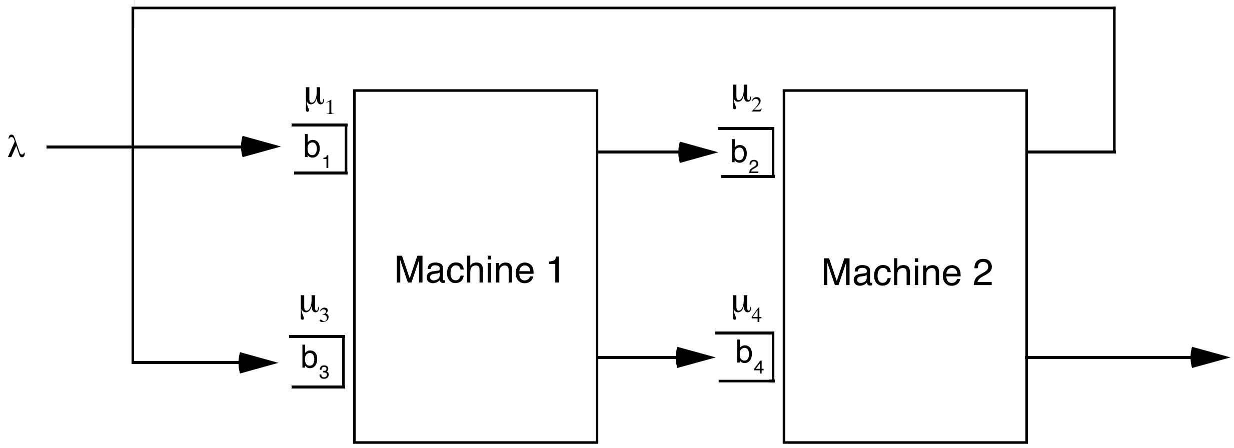

Figure 3: The Open Re-Entrant Line of Examples 1, 3, 4

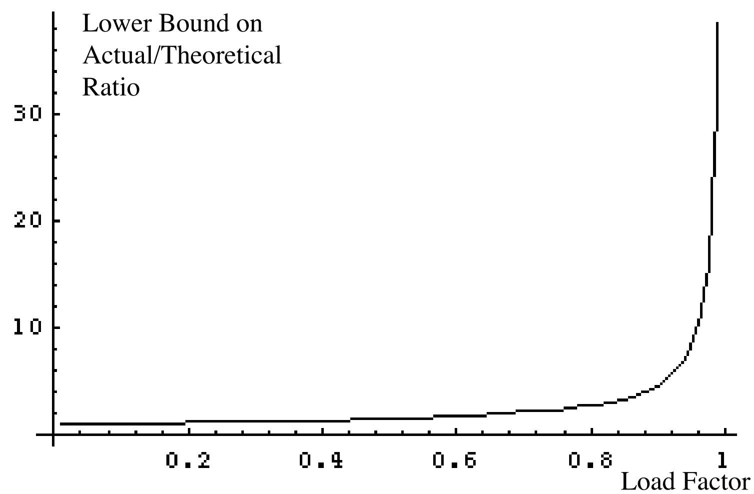

Figure 4: Lower Bound on Actual/Theoretical Ratio Versus Load Factor for Example 1

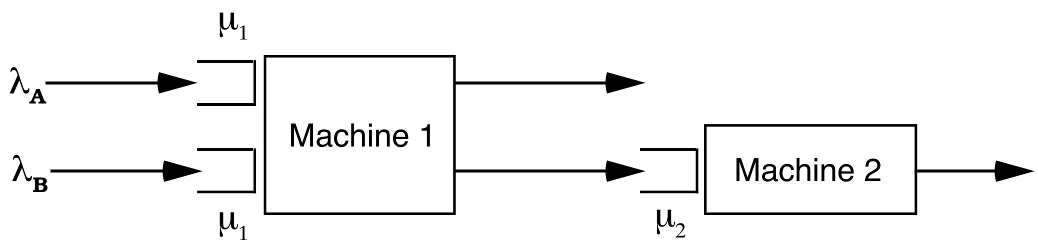

Figure 5 (a): Ou and Wein's System (a) of Example 2

Balanced: μ_1=2, μ_2=1. Imbalanced: μ_1=2, μ_2=1.5.
Light: λ_A= λ_B=0.3. Medium: λ_A= λ_B=0.6. Heavy: λ_A= λ_B=0.9. Very Heavy: λ_A= λ_B=0.99.

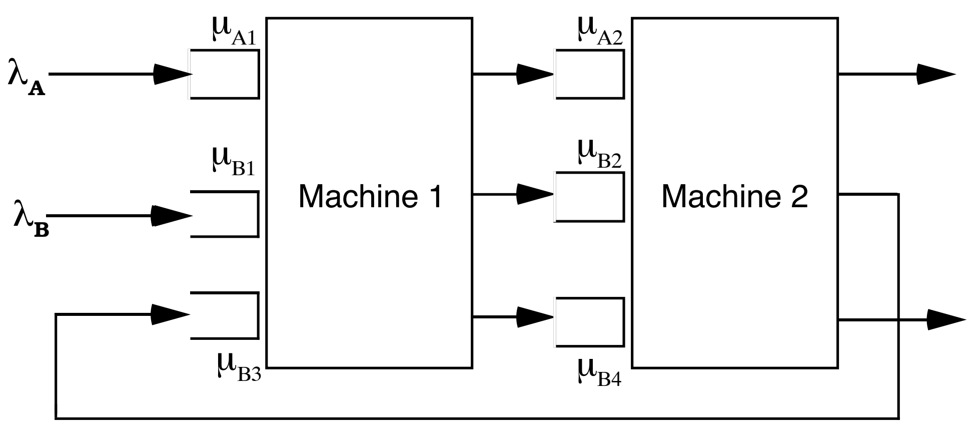

Figure 5 (b): Ou and Wein's System (b) of Example 2

Balanced: μ_{A1}=1/4, μ_{A2}=1, μ_{B1}=1/8, μ_{B2}=1/6, μ_{B3}=1/2, μ_{B4}=1/7.
Imbalanced: μ_{A1}=1/4, μ_{A2}=2/3, μ_{B1}=1/8, μ_{B2}=1/4, μ_{B3}=1/2, μ_{B4}=3/14.
Light: λ_A= λ_B=3/140. Medium: λ_A= λ_B=6/140. Heavy: λ_A= λ_B=9/140. Very Heavy: λ_A= λ_B=99/1400.

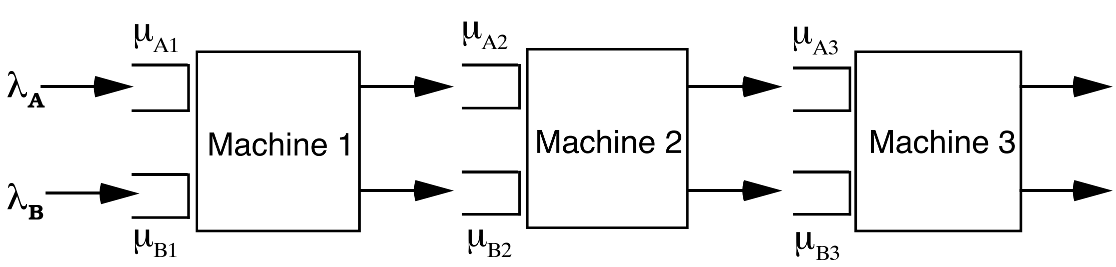

Figure 5 (c): Ou and Wein's System (c) of Example 2

Balanced: μ_{A1}=1/2, μ_{A2}=1/4, μ_{A3}=1/6, μ_{B1}=1/7, μ_{B2}=1/5, μ_{B4}=1/3.
Imbalanced: μ_{A1}=1/2, μ_{A2}=1/2, μ_{A3}=1, μ_{B1}=1/7, μ_{B3}=1/4, μ_{B4}=1/2.
Light: λ_A= λ_B=1/30. Medium : λ_A= λ_B=2/30. Heavy : λ_A= λ_B=0.1. Very Heavy : λ_A= λ_B=0.11.

System	Scenario	Ou & Wein's Bound	LP Bound
(a)	Balanced Light	0.775 (± 0.010)	0.7286
	Balanced Medium	2.47 (± 0.046)	2.10
	Balanced Heavy	13.2 (± 0.824)	9.90
	Balanced Very Heavy	85.9 (± 23.0)	99.99
	Imbalanced Light	0.621 (± 0.007)	0.6286
	Imbalanced Medium	1.85 (± 0.031)	1.90
	Imbalanced Heavy	9.46 (± 0.544)	9.60
	Imbalanced Very Heavy	72.8 (± 24.6)	99.66
(b)	Balanced Light	0.734 (± 0.011)	0.7087
	Balanced Medium	2.18 (± 0.045)	1.9385
	Balanced Heavy	8.48 (± 0.611)	8.209
	Balanced Very Heavy	43.8 (± 12.9)	72.859
	Imbalanced Light	0.56 (± 0.006)	0.624
	Imbalanced Medium	1.47 (± 0.031)	1.7562
	Imbalanced Heavy	6.94 (± 0.451)	7.6289
	Imbalanced Very Heavy	43.5 (± 11.9)	72.0349
(c)	Balanced Light	1.17 (± 0.013)	1.0116
	Balanced Medium	3.32 (± 0.055)	2.6475
	Balanced Heavy	12.2 (± 0.702)	11.4
	Balanced Very Heavy	71.4 (± 22.0)	118.14
	Imbalanced Light	0.717 (± 0.008)	0.7116
	Imbalanced Medium	1.78 (± 0.023)	1.9718
	Imbalanced Heavy	8.33 (± 0.890)	8.65
	Imbalanced Very Heavy	50.9 (± 14.8)	84.4913

Table 1: Comparison of LP Bounds with Ou and Wein's Bounds for Systems (a), (b) and (c) of Example 2

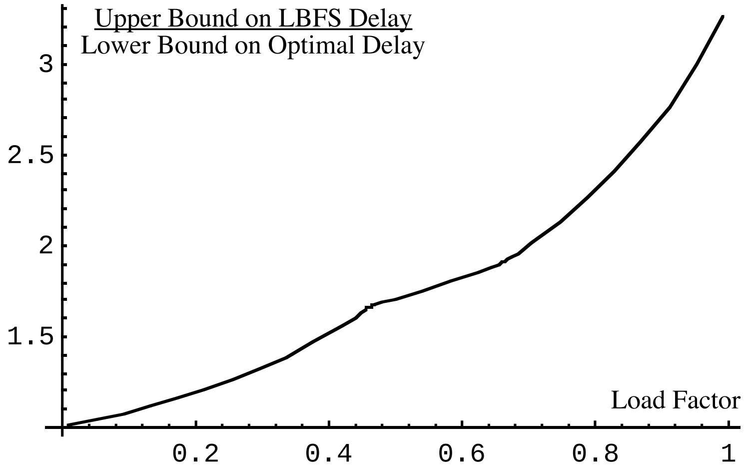

Figure 6:Upper Bound on LBFS Delay/Lower Bound on Optimal Delay in Example 3

Policy	Lower Bound on Number in System	Upper Bound on Number in System
LBFS	0.2111	0.2287
Optimal	0.2111	NA

Table 2: Near Optimality of LBFS in Example 3 at Load Factor of 0.1

Policy	Lower Bound on Number in System	Upper Bound on Number in System
LBFS	1.06667	1.60075
FBFS Bounds Without Tandem Constraints	1.6141	2.48964
FBFS Bounds With Tandem Constraints	1.73718	2.48964

Table 3: Comparison of LBFS and FBFS Policies for Example 4

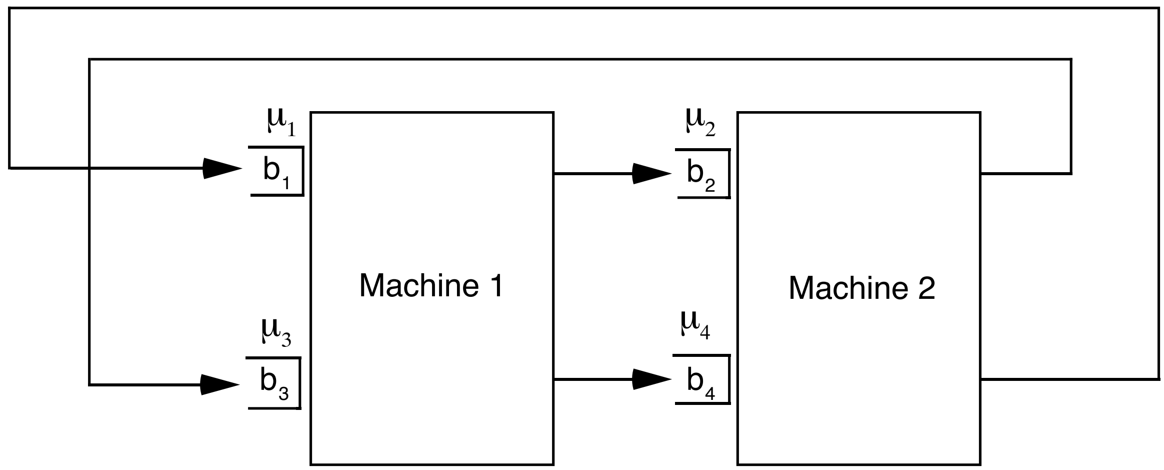

Figure 7: Closed Re-Entrant Line of Example 5

	N = 20		N = 100	
Policy	**Lower Bound on Throughput**	**Upper Bound on Throughput**	**Lower Bound on Throughput**	**Upper Bound on Throughput**
Optimal*	0.195122	0.240343	0.236686	0.248007
LBFS	0.237624	0.239282	0.247423	0.247780
FBFS	0.195122	0.240343	0.236686	0.248007
Balanced	5/21 ≈ 0.238095	0.240343	25/101 ≈ 0.247525	0.248007
Unbalanced	0.195122	5/21 ≈ 0.238095	0.236686	25/101 ≈ 0.247525

Table 4: Comparison of Policies for Closed Re-Entrant Line of Example 5

*By upper or lower bound on throughput for optimal policy we mean the upper or lower bound on the performance of any non-idling policy.

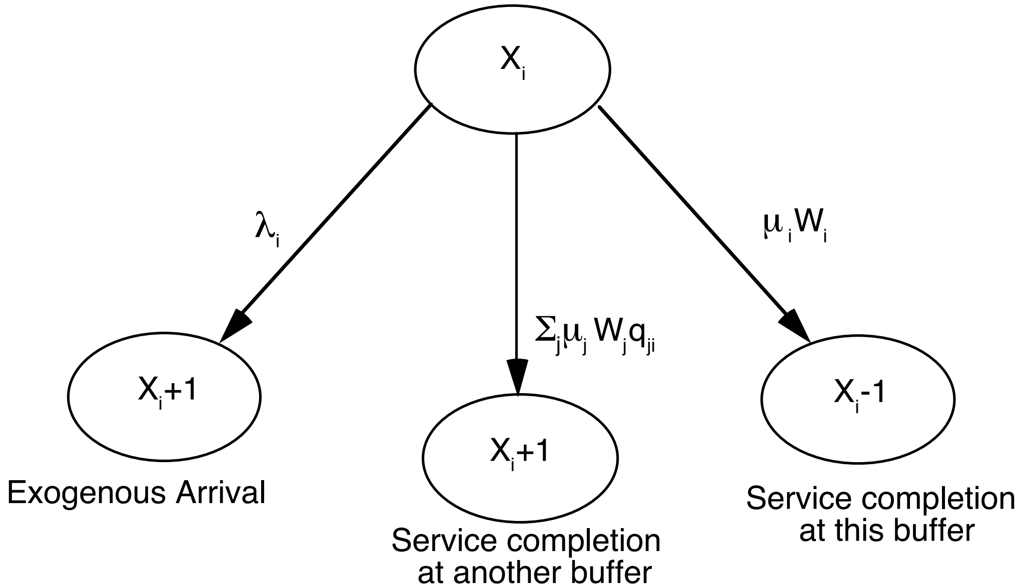

Figure 8: State Transitions Under Bernoulli Splitting

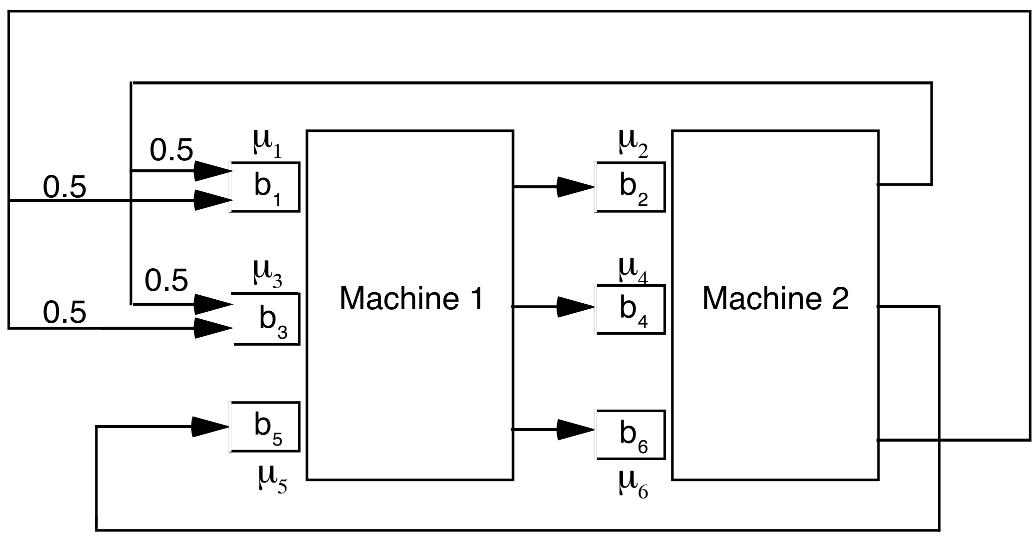

Figure 9: Harrison and Wein's Network of Example 7

Population	**Lower Bound on Throughput of Balanced Policy**	**Upper Bound on Throughput of Balanced Policy**	**95% Confidence Interval of Harrison and Wein**
7	0.125000	0.133697	(0.1261, 0.1279)
30	0.138249	0.140609	N/A

Table 5: Throughput Bounds for Harrison and Wein's Network of Example 7

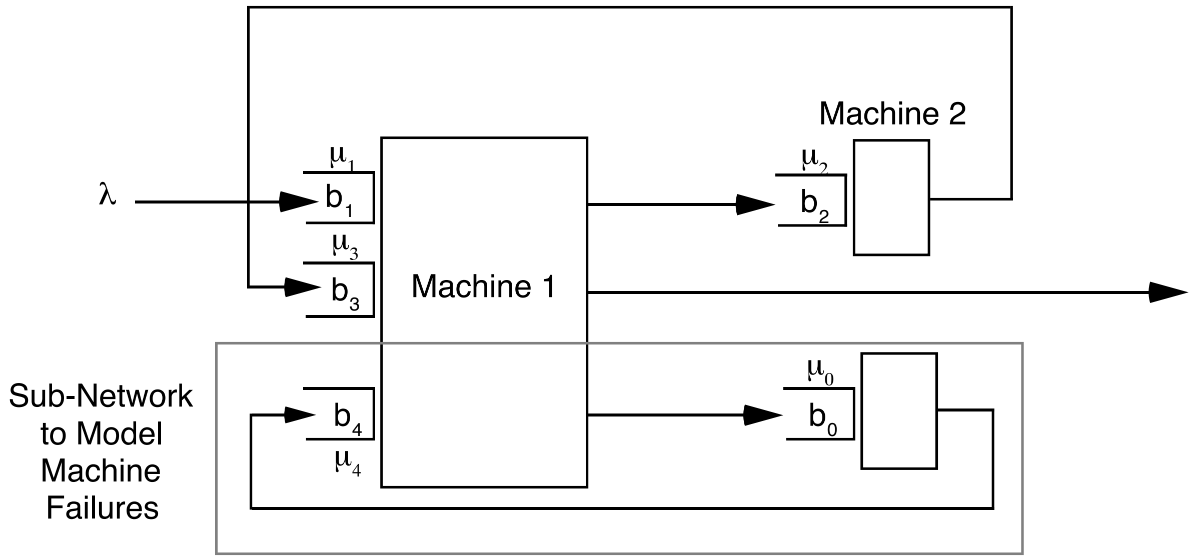

Figure 10: Model of System with Machine Failures in Example 8

MTTR/MTTF	Lower Bound on Number in System	Upper Bound on Number in System
0.2	1.5935	2.3482
0.3	1.9233	2.9356
0.4	2.3575	3.5364
0.5	2.9583	4.1825
0.6	3.8506	5.1354
0.99	174.5401	224.3769

Table 6: Effect of MTTR/MTTF on Mean Number in System of Example 8
Under the LBFS Policy

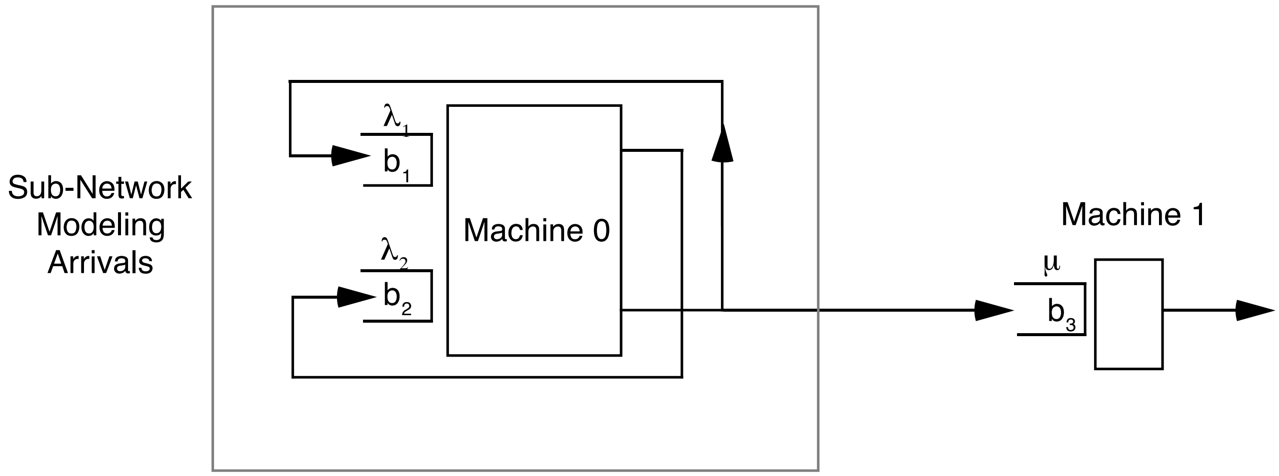

Figure 11: E_2 /M/1 Queue of Example 9

Load Factor ρ	Exact Value	Lower Bound by LP	Upper Bound by LP	King-man's Upper Bound
0.6	1.1901	0.875	1.375	1.675
0.75	2.3229	2.0000	2.5	2.875
0.9	6.8295	6.5	7.0	7.45
0.99	74.333	74.0	74.5	74.995

Table 7 : Mean Number in E_2 /M/1 Queue in Example 9