Fluctuation smoothing policies are stable for stochastic re–entrant lines*†

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Abstract

We establish that the Fluctuation Smoothing Policies for Mean Cycle Time (FSMCT) are stable for stochastic re–entrant lines. These constitute the first analytical results for stochastic systems on this class of scheduling policies, which were proposed in [1] and [2], and shown to substantially reduce the mean and variance of cycle times semiconductor plant models. A special case is the Last Buffer First Serve policy. Stability of FSMCT policies is established by showing that their fluid limits empty in finite time.

1 Introduction

A key problem in semiconductor manufacturing is to reduce the mean and variance of cycle time. For this purpose, *Fluctuation Smoothing Policies for Mean Cycle Time* (FSMCT) were proposed in [1], [2], which attempt to reduce mean cycle–time by smoothing the fluctuations in all internal flows in the system. This is accomplished by regulating the cumulative downstream shortfalls of These policies perform very well in the extensive simulation studies reported in [1].

A special case of FSMCT policies is the *Last Buffer First Serve* (LBFS) buffer priority policy, where buffers are prioritized according to the reverse of the order in which they are visited.

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While the stability of FSMCT and other Least Slack Policies was established for deterministic models in [3], no such analytical results were available for stochastic systems.

Recently, Dai [4] has shown that if the fluid limits of a queueing network empty in finite time and stay empty, then the network is stable in the sense of positive Harris recurrence. He has also used this technique to prove the stability of the First Buffer First Serve (FBFS) policy.

Here, we use this technique to establish the stability of FSMCT policies, under fairly general assumptions. We analyze the fluid limits by identifying a sequence of bottlenecks. As a corollary, we obtain the stability of the LBFS policy. Our techniques for analyzing fluid limits constrained by their integral equations may be useful for further work in the area.

We note that Dai and Weiss [5] have simultaneously and independently obtained a proof of the stability of LBFS. We also refer the reader to [5] for other stability/instability results for some examples of systems. The results here also appeared in condensed form in [6].

2 The system

Consider a system of S machines labelled $\{1, 2, \ldots, S\}$. Parts arrive at buffer b_1 with an i.i.d. interarrival time sequence $\{\alpha(n)\}$. Upon obtaining service service at b_i from machine $\sigma(i)$, they proceed next to buffer b_{i+1}, except that after b_L they leave the system. Since one may have $\sigma(i) = \sigma(j)$ for $i \neq j$, this is called a *re-entrant line*, see [7]. Parts in b_i require i.i.d. service times $\{\eta_i(n)\}$, with mean $1/\mu_i = m_i < \infty$. We suppose that $E[\alpha(1)] = \lambda = 1/\alpha < \infty$, and that $\{\alpha(n)\}$ and $\{\eta_i(n)\}$ for $i = 1, 2, ..., L$ are mutually independent. We assume that the interarrival times are unbounded, i.e., for any $x > 0$, Prob $(\alpha(1) > x) > 0$. Finally, we suppose that[1] for some integer n and some function $p(x) \geq 0$ on $\mathcal{R}_+$ with $\int_0^\infty p(x)dx > 0$,

$$\text{Prob } \left[a \leq \sum_{i=1}^{n} \alpha(i) \leq b \right] \geq \int_a^b p(x)dx \text{ for every } 0 \leq a \leq b.$$

[1] We are grateful to Sean Meyn for pointing out the need for this additional assumption.

We assume that the *nominal load* ρ_σ on each machine σ is strictly less than 1, i.e.,

$$\rho_\sigma := \lambda \left(\sum_{\{1 \le k \le L,\, \sigma(k)=\sigma\}} m_k \right) < 1. \tag{1}$$

The FSMCT policies can be described as follows, see [2]. At any time t, machine σ should work on the first part in that non–empty buffer b_k for which

$$k = \arg\min_j \sum_{i=j+1}^{L} (Q_i(t) - \xi_i). \tag{2}$$

Here, $\xi_i \in \mathcal{R}$, for $i = 1, 2, ..., L$, are intended to be estimates of the mean number of parts in b_i, usually obtained empirically. We shall let $\{\xi_i\}$ be arbitrary real numbers, subject only to the assumption that the fractional parts of $\sum_{i=j+1}^{L} \xi_i$ are unique for each j. (This can always be met by arbitrarily small perturbations of the ξ_i's). It ensures that the minimizer in (2) is unique. The service discipline is *non–idling*, i.e., a machine cannot be idle when any of its buffers is non–empty, and is preemptive resume.

The LBFS policy can be described as follows. If $j < k$ and buffers b_j and b_k share the same machine, then priority is given to b_k. It is a special case of an FSMCT policy, and is obtained when all the ξ_i's are chosen negative.

Our main result is the following.

Main Theorem *Under the above assumptions, the Markov process representing a re–entrant line operating under an FSMCT policy is positive Harris recurrent.*

The following corollary of Theorem 1 is immediate.

Corollary 1 *Under the above assumptions, the Markov process representing a re–entrant line operating under the LBFS policy is positive Harris recurrent.*

3 Proof of the Main Theorem

The main steps of the proof are as follows:

- Obtain constraints on the fluid limits of the stochastic system.

- Show that the constraints on the FSMCT fluid limits are identical to those for the LBFS fluid limits.
- Show that the fluid limits empty in finite time, and stay empty. This is done as follows:
  - Show that the last buffer b_L empties at some finite time t_L.
  - At t_L, identify an upstream bottleneck that caused b_L to empty, and show that all the buffers downstream of that bottleneck (including b_L) remain empty for all $t > t_L$.
  - Iterate the argument, at each time identifying a new upstream bottleneck and a new finite emptying time for all buffers downstream of it.

First we introduce notation and some preliminaries from Dai [4]. Let $Q_k(t)$ denote the queue length, and $v_k(t)$ the residual service time at buffer b_k at time t. Let $u(t)$ denote the residual interarrival time at time t. Define $E^u(t) := \max\{n : u + \xi(1) + \cdots + \xi(n-1) \leq t\}$. Let $D_k(t)$ denote the number of departures from buffer b_k up to time t. Then the total number of arrivals to buffer b_k, including the initial condition, and the queue lengths are,

$$A_1(t) = Q_1(0) + E^u(t), \text{ and } A_k(t) = Q_k(0) + D_{k-1}(t), \ k = 2, \ldots, L,$$

$$Q_k(t) = A_k(t) - D_k(t).$$

Let $T_k(t)$ be the amount of time in $[0, t]$ that $\sigma(k)$ spends on b_k, i.e.,

$$T_k(t) := \int_0^t w_k(s)\, ds, \text{ where}$$

$$w_k(t) := 1_{\{\sigma(k) \text{ is working on } b_k \text{ at time } t\}}.$$

Also denote the cumulative amount of time spent on buffer b_k, and those further downstream, by machine $\sigma(k)$ in $[0, t]$ by,

$$B_k(t) := \int_0^t \sum_{\{j \geq k \& \sigma(j) = \sigma(k)\}} w_j(s)\, ds.$$

Let $I_k(t) := t - B_k(t)$ denote the time *not* spent on buffer b_k and those further downstream by machine $\sigma(k)$ in $[0,t]$. Define $Q(t) := (Q_1(t),\dots,Q_L(t))$, $\xi := (\xi_1,\dots,\xi_L)$, and

$$f_k(Q(t),\xi) := \min(1, g_k(Q(t),\xi)), \text{ where}$$

$$\begin{aligned} g_k(Q(t),\xi) &:= \sum_{\substack{\{j\geq k\ \&\\ \sigma(j)=\sigma(k)\}}} Q_j(t) \prod_{\substack{\{n<j\ \&\\ \sigma(n)=\sigma(k)\}}} \left(\sum_{i=n+1}^{j} Q_i(t) - \xi_i\right)^+ \text{ if } k > \min\{j:\sigma(j)=\sigma(k)\} \text{ and} \\ &:= \sum_{\{j:\sigma(j)=\sigma(k)\}} Q_j(t) \text{ if } k = \min\{j:\sigma(j)=\sigma(k)\}. \end{aligned}$$

Now it follows from (2) that $f_k(Q(t),\xi) > 0$ implies that the highest priority non-empty buffer at $\sigma(k)$ at time t, say b_j, is such that $j \geq k$. Thus the policy and the non–idling assumption require that machine $\sigma(k)$ work on some buffer in the set $\{b_j:\ j\geq k \text{ and } \sigma(j)=\sigma(k)\}$. So

$$\int_0^\infty f_k(Q(s),\xi)\, dI_k(s) = 0 \text{ for } k=1,...,L. \tag{3}$$

The state of the Markov process representing the system is given by

$$X(t) := (Q_1(t),\dots,Q_L(t), u(t), v_1(t),\dots,v_L(t)).$$

For $x = (q_1,...,q_L,u,v_1,...,v_L)$, define

$$|x| := \sum_{k=1}^{L}(q_k m_k + u + v_k).$$

In the sequel, we shall denote explicit dependence on an initial condition x by adding a superscript x to the variable of interest. For any function f, denote its scaled version by,

$$\bar{f}^x(t) := \frac{1}{|x|} f^x(|x|t), \text{ for } t\geq 0 \text{ and } x>0.$$

For simplicity, let us first make the additional assumption that all the service and inter-arrival time distributions are exponential. This allows us to omit u and v_k from the state $x(t)$ which then consists only of $Q(t)$. This will be relaxed later.

Lemma 1 *For almost all sample paths, for every sequence $\{x_n\}$ with $|x_n|\to\infty$, there is a subsequence $\{x_{n_l}\}$ such that as $l\to\infty$,*

$$(\bar{E}^{x_{n_l}}, \bar{D}^{x_{n_l}}, \bar{A}^{x_{n_l}}, \bar{Q}^{x_{n_l}}, \bar{T}^{x_{n_l}}, \bar{I}^{x_{n_l}}) \Rightarrow (\bar{E},\bar{D},\bar{A},\bar{Q},\bar{T},\bar{I}) \tag{4}$$

where "⇒" denotes uniform convergence on compacts. Furthermore, the limit processes, called the fluid limits, satisfy:

$$\bar{E}(t) = \lambda t, \tag{5}$$

$$\bar{D}_k(t) = \mu_k \bar{T}_k(t), \tag{6}$$

$$\bar{A}_1(t) = q_1 + \bar{E}(t), \tag{7}$$

$$\bar{A}_k(t) = q_k + \bar{D}_{k-1}(t),\ k \geq 2, \tag{8}$$

$$\bar{Q}_k(t) = \bar{A}_k(t) - \bar{D}_k(t), \tag{9}$$

$$\bar{I}_k(t) = t - \sum_{\{j: k \leq j \leq L \ \& \ \sigma(j)=\sigma(k)\}} \bar{T}_j(t), \tag{10}$$

$$0 \leq \bar{T}_k(t) \leq \bar{I}_j(t) \leq t$$

$$\text{for all } j > k \text{ with } \sigma(j) = \sigma(k), \tag{11}$$

$$0 \leq \bar{T}_k(t_1) \leq \bar{T}_k(t_2) \text{ and}$$

$$0 \leq \bar{I}_k(t_1) \leq \bar{I}_k(t_2) \text{ if } t_1 \leq t_2, \tag{12}$$

$$\int_0^\infty \min\Big(\sum_{\substack{\{j: k \leq j \leq L \ \& \\ \sigma(j)=\sigma(k)\}}} \bar{Q}_j(t),\, 1\Big)\, d\bar{I}_k(t) = 0, \tag{13}$$

$$|\bar{I}_k(t) - \bar{I}_k(s)| \leq |t-s|,$$

$$|\bar{T}_k(t) - \bar{T}_k(s)| \leq |t-s|, \text{ and}$$

$$\bar{Q}_k(t) \text{ is also Lipschitz}, \tag{14}$$

$$\text{and } \sum_{k=1}^{L} m_k q_k \leq 1. \tag{15}$$

Remark The key result is (13) which asserts that in the limit, the constraints on FSMCT policies are the same as for the LBFS policy.

Proof The existence of a convergent subsequence follows from Theorem 4.1 of Dai [4], Equation (5) from Lemma 4.2 of [4], and (6) from the proof of Corollary 4.2 of [4]. Equations (7–9) are the queue length variants of (4.18–4.19) of Theorem 4.1 of [4]. Equations (10–12) are obtained trivially from the fact that they hold along sample paths by definition. Equation

(14) follows from (11) and (5–9). Equation (15) is a consequence of the scaling by $1/|x|$. So all we need to prove is the key result (13).

Define $\bar{\xi}^x := \frac{\xi}{|x|}$. From (3), for trajectories starting from initial conditions x_{n_l}, we have

$$\int_0^\infty f_k(\bar{Q}^{x_{n_l}}(s), \bar{\xi}^{x_{n_l}})\, d\bar{I}_k^{x_{n_l}}(s) = 0, \text{ for all } x_{n_l}, \tag{16}$$

Now f_k is a continuous bounded function on $\mathcal{R}^{2L}$. Also, $(\bar{Q}^{x_{n_l}}, \bar{\xi}^{x_{n_l}}) \Rightarrow (\bar{Q}, 0)$ in $D_{\mathcal{R}^{2L}}[0,\infty)$, $\bar{I}_k^{x_{n_l}} \Rightarrow \bar{I}_k$ in $C_{\mathcal{R}}[0,\infty)$, and $\bar{I}_k^{x_{n_l}}$ is non–decreasing for each l. So by Lemma 2.4 of Dai and Williams [8], we have that uniformly for all t in any compact subset of $\mathcal{R}$,

$$\int_0^t f_k(\bar{Q}^{x_{n_l}}(s), \bar{\xi}^{x_{n_l}})\, d\bar{I}_k^{x_{n_l}}(s) \to \int_0^t f_k(\bar{Q}(s), 0)\, d\bar{I}_k(s).$$

From the above and (16), we obtain

$$\int_0^\infty f_k(\bar{Q}(s), 0)\, d\bar{I}_k(s) = 0. \tag{17}$$

Note that (17) does not involve ξ's. So it must be the same as the integral equation obtained under the LBFS policy. Alternately, we can see that

$$f_k(\bar{Q}(t), 0) > 0 \Leftrightarrow \sum_{\{k \le j \le L,\, \sigma(j) = \sigma(k)\}} \bar{Q}_j(t) > 0.$$

So (17) becomes

$$\int_0^\infty \sum_{\{k \le j \le L,\, \sigma(j) = \sigma(k)\}} \bar{Q}_j(s)\, d\bar{I}_k(s) = 0, \tag{18}$$

thus yielding (13).

□

Having described a set of necessary conditions (5–15) for the fluid limits, we now show through a sequence of lemmas that these limits empty in finite time, and stay empty, from every initial condition q satisfying (15). For these results, we can consider the constraints on the fluid limits as having been inherited from the LBFS priority policy.

Lemma 2 *(i) $\bar{Q}_L(t_L) = 0$ for some finite $t_L > 0$ which depends only on λ and $\{\mu_k\}$.*

(ii) Let b_k be the buffer satisfying

$$k := \max\{i < L \mid \sum_{\{j:i\le j\le L \text{ and } \sigma(j)=\sigma(i)\}} m_j \ge m_L\}. \tag{19}$$

If no such i exists, take $k := 0$. Then, the buffers $b_{k+1}, ..., b_L$ are empty at time t_L.

Proof (i) Consider $\sum_{k=1}^{L} \bar{Q}_k(t)$. Suppose $\bar{Q}_L(s) > 0$ for all $s \in (0, t]$. Then $T_L(t) = t$, and

$$\sum_{k=1}^{L} \bar{Q}_k(t) = \sum_{k=1}^{L} q_k - (\mu_L - \lambda)t.$$

By (1) this means that $\bar{Q}_L(t) = 0$ for some $t \le \left(\sum_{k=1}^{L} q_k\right)/(\mu_L - \lambda)$. From (15), $t_L \le K/(\mu_L - \lambda)$, where K depends only on λ and $\{\mu_k\}$.

(ii) Recall that $\bar{Q}, \bar{I}, \bar{T}$ are Lipschitz. If $k = L-1$, there is nothing to prove as we already know that $\bar{Q}_L(t_L) = 0$. So we consider the case when $k \le L-2$. The proof is by induction. We note that for $\bar{Q}_L(t_L) = 0$, we must have

$$\bar{D}_{L-1}(t_L) - \bar{D}_{L-1}(t_L - \delta) \le \mu_L \delta, \text{ for all } \delta \in [0, \delta_0], \tag{20}$$

for suitably small $\delta_0 > 0$. Now if $\bar{Q}_{L-1}(t_L) > 0$, then by continuity $\bar{Q}_{L-1}(s) > 0$, for all $s \in [t_L - \epsilon, t_L]$ for suitable $\epsilon > 0$. Since $\sigma(k+j) \ne \sigma(L)$ for $j = 1, ..., L-k-1$, we have $\bar{D}_{L-1}(t_L) - \bar{D}_{L-1}(s) > \mu_L(t_L - s)$, since b_{L-1} satisfies the condition $m_{L-1} < m_L$, because $k < L-1$. This contradicts (20). So we have $\bar{Q}_{L-1}(t_L) = 0$, and $\bar{D}_{L-1}(t_L) - \bar{D}_{L-1}(t_L - \delta) \le \mu_L \delta$, for all $\delta \in [0, \delta_0]$.

Now assume that for some $j > k$, and all $i = j+1, ..., L$, we have $\bar{Q}_i(t_L) = 0$ and $\bar{D}_i(t_L) - \bar{D}_i(t_L - \delta) \le \mu_L \delta$, for all $\delta \in [0, \delta_0]$ for some $\delta_0 > 0$ sufficiently small. Then, we claim that these two relations also hold for b_j. Suppose not. Then either $\bar{Q}_j(t_L) > 0$, or, for every $\delta_0 > 0$, there is a $\delta < \delta_0$ such that $\bar{D}_j(t_L) - \bar{D}_j(t_L - \delta) > \mu_L \delta$. Now if $\bar{Q}_j(t_L) > 0$, then by continuity $\bar{Q}_j(s) > 0$, for all $s \in [t_L - \epsilon, t_L]$ for suitably small $\epsilon > 0$. But by the induction hypothesis, $\bar{D}_l(t_L) - \bar{D}_l(t_L - \delta) \le \mu_L \delta$ for all $l > j$, $\delta < \delta_0 \wedge \epsilon$. This implies that

$$\sum_{\{l:j<l<L \text{ and } \sigma(l)=\sigma(j)\}} \left[\bar{T}_l(t_L) - \bar{T}_l(t_L - \delta)\right] \le \left[\sum_{\{l:j<l<L \text{ and } \sigma(l)=\sigma(j)\}} m_l\right] \mu_L \delta.$$

Thus we have

$$\bar{T}_j(t_L) - \bar{T}_j(t_L - \delta) \geq \left[1 - \sum_{\{l: j<l<L \text{ and } \sigma(l)=\sigma(j)\}} m_l \mu_L\right] \delta.$$

Rearranging terms, we have

$$\bar{D}_j(t_L) - \bar{D}_j(t_L - \delta) \geq \left[\mu_L + (1 - \frac{\mu_L}{\mu_j^*})\mu_j\right] \delta, \tag{21}$$

where $\mu_j^* := \frac{1}{(\sum_{\{l: j \leq l<L \text{ and } \sigma(l)=\sigma(j)\}} m_l)}$. But by (19), $\mu_j^* > \mu_L$. Thus, we conclude that in either case, for every $\delta_0 > 0$, there is a $\delta < \delta_0$ such that $\bar{D}_j(t_L) - \bar{D}_j(t_L - \delta) > \mu_L \delta$. But by the induction assumption, $\bar{D}_{j+1}(t_L) - \bar{D}_{j+1}(t_L - \delta) \leq \mu_L \delta$, for all $\delta \in [0, \delta_0]$. This implies that $Q_{j+1}(t_L) > 0$, which is a contradiction. So $Q_j(t_L) = 0$ and $\bar{D}_j(t_L) - \bar{D}_j(t_L - \delta) \leq \mu_L \delta$, for all $\delta \in [t_L - \delta_0, t_L]$. □

Lemma 3 *Let b_k and t_L be as in the previous lemma. Then*

$$\bar{Q}_L(\delta + t_L) = \cdots = \bar{Q}_{k+1}(\delta + t_L) = 0, \text{ for all } \delta \geq 0.$$

Proof If $k = 0$, define $\bar{D}_k(t) := \lambda t$. Otherwise $\bar{D}_k(t)$ is as in (4). Suppose $\bar{Q}_L(t) > 0$ for some $t > t_L$. We know $\bar{Q}_L(t_L) = 0$. So, $\bar{D}_{L-1}(t) - \bar{D}_{L-1}(t_L) > \mu_L(t - t_L)$. But $\bar{Q}_{L-1}(t_L) = 0$. So $\bar{D}_{L-2}(t) - \bar{D}_{L-2}(t_L) > \mu_L(t - t_L)$, and so on. Thus, we can conclude that $\bar{D}_j(t) - \bar{D}_j(t_L) > \mu_L(t - t_L)$, for all $j = k, ..., L-1$. If $k = 0$, we immediately obtain a contradiction. If $k > 0$, this implies that

$$\sum_{\{l: k \leq j \leq L \text{ and } \sigma(l)=\sigma(j)\}} \left[\bar{T}_l(t) - \bar{T}_l(t_L)\right] > \left[\sum_{\{l: k \leq j \leq L \text{ and } \sigma(l)=\sigma(j)\}} m_j\right] \mu_L(t - t_L).$$

But the right hand side above is at least as large as $(t - t_L)$, by (19). This is a contradiction. So $\bar{Q}_L(t_L + \delta) = 0$, for all $\delta > 0$.

If $k = L - 1$, there is nothing else to prove. So assume that $k < L - 1$. Assume for induction that $\bar{Q}_L(t_L + \delta) = \cdots = \bar{Q}_{j+1}(t_L + \delta) = 0$, for all $\delta > 0$, for some $j > k$. For this, by (6,7), we require that

$$\bar{D}_l(t) - \bar{D}_l(t - \delta) \leq \mu_L \delta, \tag{22}$$

for all $l = j, ..., L$, all $t > t_L$ and $0 < \delta < t - t_L$. Now if for some $t > t_L$, $\bar{Q}_j(t) > 0$. Then arguing exactly as we did while obtaining equation (21), we get $\bar{D}_j(t) - \bar{D}_j(t-\delta) > \mu_L \delta$, which is a contradiction to (22). So the proof is complete by induction. $\square$

Proof of the Main Theorem In the preceding lemmas we have identified a time t_L at which the buffers $b_{k+1}, \ldots, b_L$ are all empty, and remain empty thereafter. If we now show that there exists a time $t_k > t_L$, which depends only on λ and $\{\mu_i\}$, such that b_k empties and remains empty thereafter, then we are done by iteration, since there are only L buffers. Now the existence of such a t_k follows analogously to Lemma 2 (i) as follows. Suppose $\bar{Q}_k(s) > 0$, for all $s \in (t_L, t_L + \delta]$. Then we have,

$$\bar{T}_k(t_L + \delta) - \bar{T}_k(t_L) = \delta - \sum_{\{i: k < i \leq L \text{ and } \sigma(i) = \sigma(k)\}} \left[\bar{T}_i(t_L + \delta) - \bar{T}_i(t_L)\right]. \tag{23}$$

But from Lemma 3, we can conclude that $\bar{D}_i(t_L + \delta) - \bar{D}_i(t_L) = \bar{D}_k(t_L + \delta) - \bar{D}_k(t_L)$, for all $i = k+1, ..., L$. Using (4) and (23), we get

$$\bar{D}_k(t_L + \delta) - \bar{D}_k(t_L) = \frac{\delta}{\sum_{\{i: k \leq i \leq L \text{ and } \sigma(i) = \sigma(k)\}} m_i}.$$

Thus we conclude that

$$0 \leq \sum_{l=1}^{L} \bar{Q}_l(t_L + \delta) = \sum_{l=1}^{L} \bar{Q}_l(t_L) + \left(\lambda - \frac{1}{\sum_{\{i: k \leq i \leq L \text{ and } \sigma(i) = \sigma(k)\}} m_i}\right) \delta.$$

So $\bar{Q}_k(t_k) = 0$ for some

$$t_k \leq \frac{\sum_{l=1}^{L} \bar{Q}_l(t_L)}{\left(\frac{1}{\sum_{\{i: k \leq i \leq L \text{ and } \sigma(i) = \sigma(k)\}} m_i} - \lambda\right)}$$

But $\sum_{l=1}^{L} \bar{Q}_l(t_L) < \lambda t_L + \sum_{l=1}^{L} q_l < K\left(1 + \frac{\lambda}{(\mu_L - \lambda)}\right)$ by (15) and Lemma 2 (i).

At time t_k we next identify a k' such that

$$k' = \max\{j < k \mid \sum_{\{i: j \leq i \leq L \text{ and } \sigma(i) = \sigma(j)\}} m_i \geq \sum_{\{n: k \leq n \leq L \text{ and } \sigma(n) = \sigma(k)\}} m_n\}.$$

This is the bottleneck which caused b_k to empty. Then we repeat the arguments of Lemmas 3 and 4 to show that $b_{k'+1}, \ldots, b_k$ are empty at t_k and remain empty thereafter.

We can relax the exponential distribution assumptions by using Theorem 5.2 of Chen [9]. Finally, we apply Theorem 4.3 of Dai [4] to conclude positive recurrence. □

4 Concluding remarks

The results on the stability of the class of FSMCT policies and the LBFS policy provide the useful extension of the stability results for bursty deterministic models in [3] to the stochastic setting. Finally, the techniques for deducing the behavior of the fluid limits of [4] from the integral equations may be useful in future work.

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