Racing with the Sun: The Optimal Use of the Solar Power Automobile[1] [2]

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Abstract

In the summer of 1997, the University of Illinois competed in Sunrayce '97, a ten-day race of solar powered vehicles across the midwestern United States. This paper provides race strategy results obtained by optimal deterministic and stochastic control techniques.

1 Introduction to Sunrayce

June 1997 marked the fourth running of Sunrayce, a ten-day cross-country race of solar powered vehicles. Thirty-six solar cars from different universities raced from Indianapolis to Colorado Springs. The University of Illinois' entry into Sunrayce was the "Photon Torpedo". Due mostly to reliability problems, the Photon Torpedo finished a disappointing fourteenth in this year's race.

Sunrayce resembles the Tour de France in that each team races against the clock. Each team is started at a different time and timed separately as the race progresses. The race consists of nine racing days (and one rest day) with each day's route running about 150 miles. Scoring for each day's leg is the time taken to complete that day's course plus time for any penalties that are assessed. For example, the penalty for not crossing the finish line within eight hours of a team's start time is scored as eight hours plus three minutes for each mile not covered on that day's race.

The primary power source for a solar car is, of course, solar power from photovoltaic cells. Part of the time, the current from the array is completely devoted to powering an electric DC motor. However, each car is also allowed to have several lead-acid batteries. Once the race has begun, only solar power may be used to charge them.

Battery power is essential to success. They provide the excess current needed while accelerating, climbing hills, or just traveling at a high speed. In addition, batteries are heavily used on cloudy days. The batteries are charged during racing if the solar power exceeds the car's power requirements, but most battery charging occurs while static charging. This entails removing the solar array from the car and pointing it directly at the sun to charge up the batteries. Before and after each day's leg, teams are allowed to static charge. Of course, this assumes that the team finishes that day's leg before sundown. Teams may also static charge during the rest day.

Once the solar car has been built and tested, the question arises about how to race with it. This was also investigated by General Motors for the 1987 World Solar Challenge[1]. Since most teams are extremely energy limited for the race, strategy becomes critical if one is to win. We need to determine at what speed we should be traveling during all parts of the race. In what way should we optimally accelerate from a stop? Should we cruise at a constant speed, at a constant power, or in some other way? How should we climb and descend hills?

2 Dynamics of the Solar Car

2.1 Power Lost and Supplied

Rolling resistance draws power linearly as the car moves at faster velocities: $P_{roll} = mgVC_r$ where V is the car's velocity and C_r is the coefficient of rolling resistance generally given in (lbs of resistive force)/(1000 lbs of car weight). The power lost due to a road grade is $P_{grade} = mgzV$ where z is the road grade (i.e., the slope of the road). Finally, and ultimately most importantly, the car also experiences aerodynamic loss which is *cubic* in velocity:

$$P_{aero} = .5\rho_{air} C_d A_f V^3 \qquad (1)$$

where ρ_{air} is the density of air, C_d is the coefficient of aerodynamic drag, and A_f is the frontal area of the car. This means a slight increase in velocity can lead to a dramatic increase in power losses due to aerodynamic

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drag. Because of this nonlinearity with respect to velocity, racing strategy is critical. For example, if today is sunny and tomorrow is cloudy, a team could travel at a high velocity today, but be forced to a very slow velocity tomorrow due to energy limitations. However, if that team were to travel more slowly today, they could save their energy and travel at a higher velocity tomorrow achieving a *lower total time*!

The total power needed to propel a solar car at a *constant* velocity is,

$$P(V) \quad = \quad P_{aero} + P_{grade} + P_{roll}. \qquad (2)$$

The power required to *accelerate* is $P_{accel} = mAV$ where A is the instantaneous acceleration. Consequently, the power that is seen at the output of the wheel is $P_{out} = mAV + P(V)$.

Power is supplied from two sources: the solar array and the batteries. Solar power $P_s(t)$ and battery current $I_B(t)$ will be modeled as arbitrary functions of time. The battery voltage is $E_B[I_B(t)] = E_0 - RI_B$ where R is the internal resistance. The power supplied to the solar car is, $P_{supplied} = P_s(t) + I_B(t)E_B(I_B)$. It should be noted that solar power could also be modeled as a function of position, $P_s(x)$.

2.2 Motor and Drive Chain Power Losses

The drive chain efficiency is assumed to be a constant, α. The motor efficiency is a function of the motor's rpm and $P_{supplied}$. The Photon Torpedo used a Unique Mobility motor so the efficiency functions used in this paper are representative of that motor. By studying the efficiency curves for this motor over a variety of motor rpm's, it was determined that, for positive powers, the relationship between $P_{supplied}$ and P_{out} was well modeled by 2 intersecting straight lines. However, these two lines would be different at every value of the rpm. Therefore, linear interpolation was used to construct efficiency lines between the known data.

In addition to positive power, most solar cars are equipped with regenerative braking systems which turn the motor backwards, acting as a generator to slow the car and charge the batteries. However, since this power effectively goes "through" the chain drive and motor, it is also subject to the same efficiency losses.

2.3 Battery Model

Batteries are not ideal energy sources. It was estimated that at a current of i amps, the equivalent energy would be that of drawing $i \times (1 + .01 \times i)$ amps in an ideal battery.

2.4 The System Equations

The system's power balance equation is

$$\alpha\eta_{motor}[P_s(t) + I_B(t)E_B(I_B)] = P(V) + mAV$$

where η_{motor} is the motor efficiency. Solving for the acceleration A we get,

$$A(t) = \frac{\alpha\eta_{motor}[P_s(t) + I_B(t)E_B(I_B)] - P(V(t))}{mV}.$$

But since

$$\begin{aligned} P_{out} &= \alpha\eta_{motor}[P_s(t) + I_B(t)E_B(I_B(t))] \\ &= f(P_s(t) + I_B(t)E_B(I_B(t)), V(t)), \end{aligned}$$

one can interpolate from the motor efficiency curves and write the state equations as,

$$\frac{dV(t)}{dt} = \frac{P_{out}(P_s(t) + I_B(t)E_B(I_B), V(t)) - P(V(t))}{mV},$$

$$\frac{dx(t)}{dt} = V(t),$$

where $x(t)$ is the position of the solar car over time.

3 Formulation of the Problem

The input to the system is the battery current $I_B(t)$ and the output is the car's velocity $V(t)$. The ending state of the car is essentially described by four variables: the final position of the car, its final velocity, the final charge remaining in its batteries, and the final time.

One may optimize with respect to any of the above four quantities, *subject to holding the other three fixed.* Indeed any optimal solution in the racing sense of the term necessarily has to be optimal with respect to any of the above four choices. We will consider the variant where one fixes the the final position, final velocity, and final time. The problem considered is to minimize the charge

$$\int_0^T (1 + .01\ I_B)I_B\ dt \qquad \text{(Ampere-seconds)}$$

needed to satisfy the terminal state requirements.

We use the Pontryagin Maximum Principle [2]. The Hamiltonian is,

$$\begin{aligned} H(I_B, V, x) = {} & \lambda_2 V + I_B(1 + .01 I_B) \\ & + \lambda_1 \tfrac{(P_{out}(P_s(t)+I_B(t)E_B(I_B(t)), V(t)) - P(V)}{mV} \end{aligned}$$

where λ_1 and λ_2 are the usual Lagrange multipliers. We thus need to solve the Two Point Boundary Value Problem

$$\begin{aligned} \tfrac{d\lambda_1}{dt} = {} & \lambda_1 \tfrac{P_{out}(P_s(t)+I_B(t)E_B(I_B(t)), V(t)) - P(V)}{mV^2} \qquad (3) \\ & -\lambda_1 \tfrac{P'_{out}(V) - P'(V)}{mV} - \lambda_2, \end{aligned}$$

$$\frac{d\lambda_2}{dt} = \lambda_1 P'(x) = \lambda_1 g z'(x),$$

where, including the two state equations given above, we have four differential equations. We thus need four boundary conditions, which are given by the initial as well as terminal conditions on the two state equations.

4 Solving the Differential Equations

To solve the TPBVP, a shooting method has been employed. We search for appropriate *initial conditions* on the remaining two variables λ_1 and λ_2 until we obtain satisfaction of the desired terminal constraints on $V(T)$ and $x(T)$. From the maximum principle, $I_B(t)$ is chosen to minimize the Hamiltonian at each time t. The model is extremely sensitive to $\lambda_{1,init}$. This precision creates a problem in that in most cases, the model will not converge for a problem with a time horizon longer than 15 minutes.

5 Maximum Principle Results

The following are descriptions of various situations of particular interest to a solar car racing team.

5.1 Constant Conditions

This is the problem of driving under constant conditions. It is desired that the car start initially at 17 m/s and terminate at the same velocity in 1200 seconds and in 20400 m. The question is whether it is ideal to maintain a constant speed throughout the run or to vary it in some way. Since it is suspected that variations in speed are inherently lossy, we expect a constant speed solution. In fact, this is the result that the maximum principle model delivers, a constant speed of 17 m/s and a constant current of 9.166 A.

5.2 Acceleration from a Stop

Consider the problem of accelerating from a stop to a given cruising velocity. This is to occur in 250 seconds and in 3817 m. The optimal solution can be seen in Figure 1. The optimal velocity profile shows a fairly constant acceleration of about 0.3 m/s^2 for 50 seconds and then the profile smoothly rounds off the curve and proceeds at a constant 17 m/s for the rest of the allotted time. To achieve this, the current profile spikes up to close to 20 A in the first 10 seconds but then settles down to the steady state value of 9.26 A.

5.3 Deceleration to a Stop

Two different scenarios will be studied. First, consider a scenario in which the driver has ample warning that he must stop. The driver has 150 seconds and 855 m to stop from a velocity of 17 m/s. Here the optimal

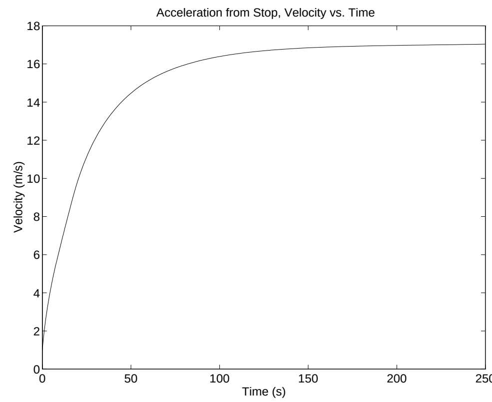

Figure 1: Acceleration from stop, velocity.

solution is to use regenerative braking for about the first 40 seconds, until the velocity is reached where the driver can apply virtually zero current and the car will come to a rest at the desired position and time (see Figures 2 and 3).

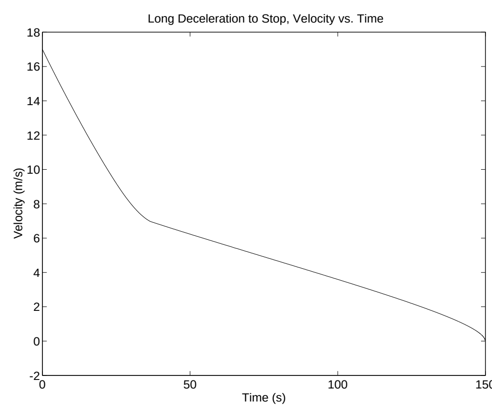

Figure 2: Long deceleration to stop, velocity.

For the other stopping situation mentioned, the driver is given only 30 seconds and 300 m to stop from 17 m/s. In this solution, as one would suspect, the optimal solution applies regenerative braking for the entire 30 seconds to stop in time.

5.4 Climbing Hills

One of the main questions in dealing with energy management is how to climb and descend hills. To answer the question of climbing a hill, a road profile was studied that starts out flat for 500 m, increases to a 2% grade from 500 to 1000 m, and stays at this grade until 1500 m. The profile then decreases its grade until it is 0% at 2000 m and remains that way until the end. With an initial and final velocity of 17 m/s, a time horizon of 300 seconds, and a distance of 4995 m, the optimal results are shown in Figure 4.

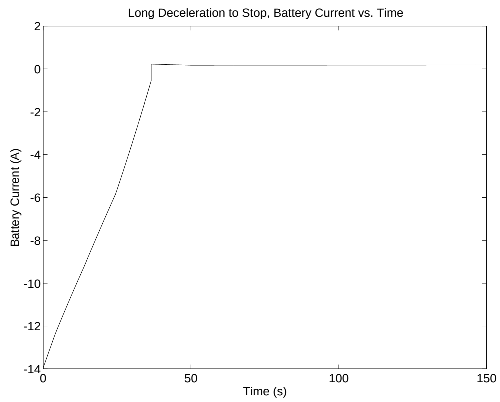

Figure 3: Long deceleration from stop, current.

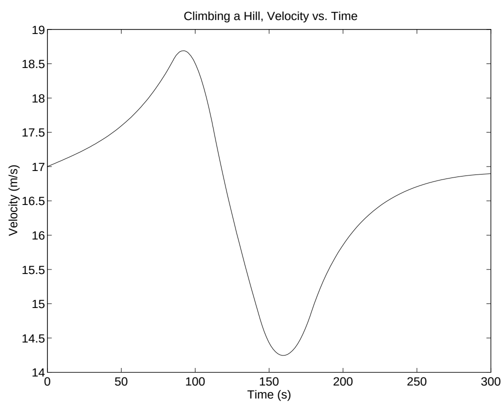

Figure 4: Ascending hill, velocity.

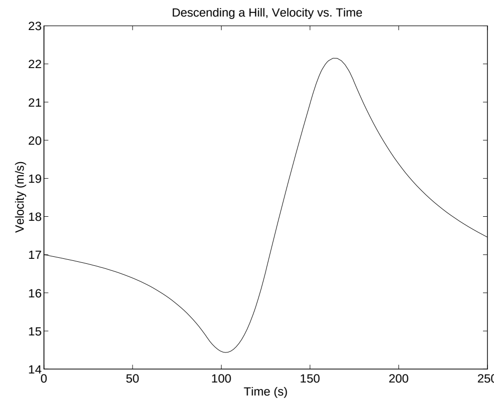

Figure 5: Descending hill, velocity.

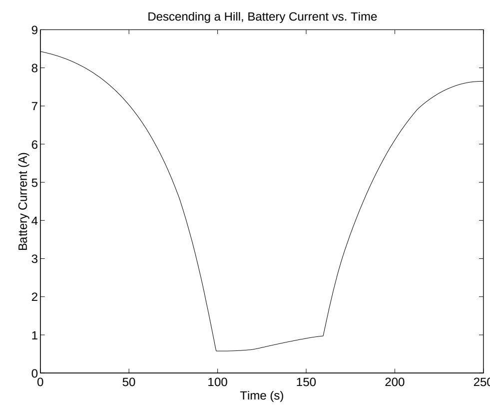

Figure 6: Descending hill, current.

This solution suggests increasing velocity in anticipation of the hill and allowing gravity to slow the car as it ascends by decreasing the current. The current is decreased so that it will reach the steady state value for traveling at 17 m/s on flat ground and, once on top of the hill, the car will accelerate back to 17 m/s.

5.5 Descending Hills

The profile used to study descending hills begins with a flat portion for 1500 m, decreases to a -3% grade from 1500 to 2000 m, and remains at this down grade until 2500 m. Then it increases its grade back to 0% at 3000 m and stays this way until the end. The time horizon was 250 seconds, and the distance 4431 m with an initial and final velocity of 17 m/s. The optimal solutions can be seen in Figures 5 and 6. The optimal velocity profile generated is very much opposite of the one generated for ascending a hill. It advises first slowing down in anticipation of the hill and then coasting, allowing gravity to accelerate the car as it proceeds down the hill. It then increases its current back to the flat ground steady state level associated with a speed of 17 m/s. Hence one does not use regenerative braking unless it is necessary to keep the car from exceeding the speed limit.

5.6 Non-constant Solar Power

A very important strategic question is how to most efficiently use the solar power input. We treat the solar power as a function of position rather than time. For example, if the car is currently getting 300 W of incident radiation from the sun, but it is known that in 4000 m the sun's radiation will increase to 800 W, should the driver increase speed to get out from under the cloud and charge on the other side, or decrease speed now with the expectation that he can go faster once he passes 4000 m? This sun intensity profile is used on flat ground in which the initial and terminal velocities are each 17 m/s. The time is 500 seconds for a distance of 8125 m. The velocity plot from the maximum principle solution is shown in Figure 7.

The solution suggests that the driver should slow down while under the cloud and increase speed again once he is clear of it. So it is more beneficial to use the solar power while one has it than to store it in the batteries.

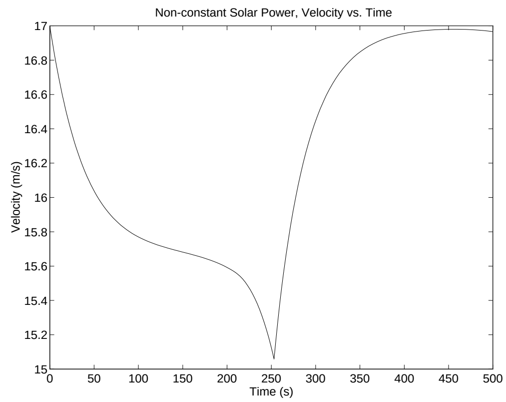

Figure 7: Changing solar power, velocity.

5.7 Adjusting the Final Position

Consider a desired velocity profile in which the average velocity is higher than either the initial or terminal velocities. This can be accomplished by setting $\dot{\lambda}_{1,final} = 0.1$. The initial velocity is set to 17 m/s and the terminal velocity is 17.34 m/s. The run is 200 seconds with an ending position of 3522 m. The average velocity is 17.61 m/s. This can be seen in Figure 8. Conversely, by setting $\dot{\lambda}_{1,final} = -0.1$, one can acquire a velocity profile in which the average velocity is smaller than the initial and final velocities.

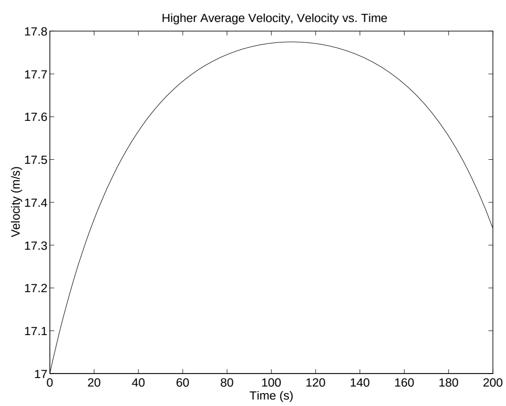

Figure 8: $\dot{\lambda}_1 = 0.1$, velocity.

6 Comparisons of Other Strategies to Maximum Principle Results

6.1 Constant Conditions

The maximum principle solution for a flat ground problem in which the initial velocity equals the final velocity is a constant velocity profile. For a 17 m/s initial and final velocity with an ending position of 8517 m and time of 500 seconds, the cost for the maximum principle solution is 5003 A-s. To determine the effects of varying this velocity, we compare this with a velocity profile which varies above and below this constant velocity. This roughly models the effect of using the driver's foot to control speed rather than a cruise control. We will therefore use a velocity profile that starts at 17 m/s for 100 seconds, accelerates to 20 m/s in 50 seconds, and spends 50 seconds at this velocity. The profile next decelerates to 14 m/s in 100 seconds, spends 50 seconds at this velocity, and accelerates back to 17 m/s. The cost for this profile is 5150 A-s, so the constant velocity solution represents a 3% cost savings.

6.2 Acceleration from a Stop

The optimal solution for accelerating from rest (or close to it), is a smooth curve (Figure 1) which can be closely approximated by two straight lines, one of constant finite acceleration and one of constant velocity. We will compare solutions for accelerating from 1 to 17 m/s in 250 seconds and in 3817 m. The maximum principle solution, requires a cost of 3036 A-s. The equivalent straight line solution, with the first line accelerating at 0.2953 m/s^2 for 54.175 seconds and zero acceleration for the remaining 195.825 seconds, requires a cost of 3134 A-s.

Another possibility is to accelerate more quickly at first, say at 0.35 m/s^2 for 43.82 seconds, and then use a small positive acceleration (0.0032 m/s^2) for the remaining time. The cost for this is 3115 A-s. Alternatively, one may accelerate more slowly at the beginning, 0.2 m/s^2 for 92.15 seconds, and decelerate at -0.015 m/s^2 for the remaining 157.85 seconds to meet the problem requirements. The cost for this scenario is 3251 A-s. In this case, one can achieve about a 7% energy savings by using the maximum principle solution.

6.3 Hill Climbing

When climbing a hill, the optimal solution accelerates in anticipation of the hill, decelerates while climbing the hill, and speeds back up once the summit is reached (Figure 4). Another strategy is a constant current strategy. When one applies a constant current of 12.72 A, the same position, 4995 m, is reached in 300 seconds. The cost of the optimal solution is 4061 A-s. On the other hand, the cost associated with the constant current solution is 4303 A-s. Therefore, the maximum principle solution gives a 6% energy savings over the constant current strategy.

7 Dynamic Programming for Weather Strategy

In order to create long term driving strategies in which weather would be considered, a dynamic programming approach was utilized [3]. The goal is to minimize the total time needed to complete the race course. A detailed model [4] was used to determine the driving time for each leg given a particular energy budget for that day and various weather scenarios. The following equation was used to determine the array of possible times at each stage for discrete energy uses in that stage,

$$T_n(C_n) = \sum_W [P_n(W) \times \mathrm{Min}_{C_{n+1}}[D_n(C_n - C_{n+1}, W) + T_{n+1}(C_{n+1})]],$$

where $T_n(C_n)$ is the expected remaining race times starting from day n and C_n is the charge state on that day. $D_n(C_n - C_{n+1}, W)$ is the time taken on day n with weather W, an initial charge C_n, and the next day's starting charge, C_{n+1}. In addition, $P_n(W)$ is the probability that weather scenario W will occur on day n. Once the user knows the current energy charge and has a nearly 100 percent weather prediction for that day, he can use tomorrow's T array to determine how the car should drive today based on the given weather probabilities for the following days.

8 Post-Race Analysis

In order to make good strategic decisions, it is imperative that the team have a good model of the vehicle. This became a major problem for the Photon Torpedo team because of insufficient testing time. Therefore, at the time of the race, the team thought that their car was more competitive than it actually was. In limited testing done before the race, the drag was estimated to be about 0.15, and this was the value used throughout the race in making driving decisions. But, it is now estimated that the drag was actually about 0.18.

Following is a day by day analysis of how the University of Illinois *should* have run the first half of the race given the predicted weather and no break downs.

Day 1: On day 1, we cruised at 55 mph but made a mistake on our hill strategy. We slowed down on the ascent and sped up on the descent. However, since day 1 was so short and the weather was relatively sunny, the hills should have been driven more aggressively because there was plenty of energy available.

Day 2: On day 2, the dynamic programming strategy suggests that we should have used about 2 kW-hr of the battery and driven the 169 mile course in 5.08 hours. This implies an average speed of 35 mph and a cruising speed of 38 mph. This is fairly slow because of a strong headwind this day. Due to our incorrect model (and foolish optimism), we drove much too fast on this day, draining the pack.

Day 3: Day 3 was mostly cloudy during the day. Therefore the recommend driving speeds are slow, a 33 mph cruise speed. This will consume about 1 kW-hr and take 5.06 hours to complete the 165 mile course.

Day 4: Day 4 was mostly cloudy during the morning and then clear after midday. The strategy program suggests driving the 156 mile leg in 3.78 hours. This means driving at a cruise speed of 44 mph and consuming 3 kW-hr of energy. Of course, following the strategy of changing solar power suggests driving more slowly in the morning and faster in the afternoon. In the actual race, the car suffered a major array failure and was only barely able to complete this day's course.

Day 5: This was the rest day. It was sunny so full recharging is easily accomplished.

Because of breakdowns and strategic mistakes, the U of I finished 16th out of 36 in the first half of the race with a time of 20.1 hours. However, this analysis says that if the proper strategy model was followed and no breakdowns occurred that the Photon Torpedo would have finished a more respectable 10th in a time of 15.3 hours. This significant improvement is caused mostly by the omission of breakdowns, but the improved strategy would have helped as well, especially given the improper use of energy on Day 2.

9 Concluding Remarks

The University of Illinois once again has high hopes for Sunrayce '99. The team members still involved in the project feel that the experience gained in this year's race will provide a strong foundation for the future of solar car racing at this university. Hopefully they can continue the work found here and use the wisdom acquired by the 1997 team to compete well in the future.

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