A correction to the proof of a lemma in “The capacity of wireless networks”

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Abstract
The proof of Lemma 4.8 in [1] is corrected.

1 Correction of proof of Lemma 4.8

In [1], Section IV.G, there was an error in the stated sample size bound for the Vapnik-Chervonenkis Theorem. The notation used will be that in [1] with $d(\mathcal{F})$ denoting the VC-dimension of the set $\mathcal{F}$. The correct statement is

$$\text{Prob} \left( \sup_{F \in \mathcal{F}} \left| \frac{1}{N} \sum_{i=1}^{N} I(x_i \in F) - P(F) \right| > \epsilon \right) \leq 4 \left[ (2N)^{d(\mathcal{F})+1} + 1 \right] e^{-\frac{2N}{n}} \text{ for } N \geq 2/\epsilon^2. \quad (1)$$

See [2].

**Proof of Lemma 4.8** The probability that a cell $V$ in $\mathcal{V}_n$ contains no nodes is $\leq \left( 1 - \frac{100 \log n}{n} \right)^n$. Since there are at most $\frac{n}{100 \log n}$ cells,

$$\text{Prob} (\text{Every cell } V \in \mathcal{V}_n \text{ contains a node}) \geq 1 - \frac{n}{100 \log n} \left( 1 - \frac{100 \log n}{n} \right)^n.$$  

Since $\lim_{n \rightarrow +\infty} n \left( 1 - \frac{\log n}{n} \right)^n = 1$, the right hand side above converges to one. □

It may be noted that this proof also allows us to use cells of smaller size, dispensing with the factor 100 in [1], equation (13). Another proof of the above Lemma is based on the probability of

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an $\epsilon$-net [3], for which the sample size given in [1] is indeed the appropriate one; however, it does not allow us to dispense with this factor.

It should also be noted that Lemma 4.13 continues to follow from (1) with $\mathcal{F} = \mathcal{D}'$ and $\epsilon = 16\sqrt{\frac{\log n}{n}}$, since $d(\mathcal{D}') \leq 30$ and $c_0$ can be chosen as 16 plus the constant in Lemma 4.9. □

Acknowledgment: The author is grateful to his student Feng Xue for noticing the misstatement.

References

