A correction to the proof of a lemma in "The capacity of wireless networks"

P. R. Kumar*

Abstract

The proof of Lemma 4.8 in [1] is corrected.

1 Correction of proof of Lemma 4.8

In [1], Section IV.G, there was an error in the stated sample size bound for the Vapnik-Chervonenkis Theorem. The notation used will be that in [1] with $d(\mathcal{F})$ denoting the VC-dimension of the set $\mathcal{F}$. The correct statement is

$$\text{Prob}\left(\sup_{F\in\mathcal{F}}\left|\frac{1}{N}\sum_{i=1}^{N} I(x_i \in F) - P(F)\right| > \epsilon\right) \leq 4\left[(2N)^{d(\mathcal{F})+1}+1\right] e^{-\frac{\epsilon^2 N}{8}} \text{ for } N \geq 2/\epsilon^2. \quad (1)$$

See [2].

Proof of Lemma 4.8 The probability that a cell V in $\mathcal{V}_n$ contains no nodes is $\leq \left(1-\frac{100\log n}{n}\right)^n$. Since there are at most $\frac{n}{100\log n}$ cells,

$$\text{Prob(Every cell } V \in \mathcal{V}_n \text{ contains a node)} \geq 1 - \frac{n}{100\log n}\left(1-\frac{100\log n}{n}\right)^n.$$

Since $\lim_{n\to+\infty} n\left(1-\frac{\log n}{n}\right)^n = 1$, the right hand side above converges to one. □

It may be noted that this proof also allows us to use cells of smaller size, dispensing with the factor 100 in [1], equation (13). Another proof of the above Lemma is based on the probability of

*Email: prkumar@uiuc.edu. Please address all correspondence to the author at Coordinated Science Laboratory, University of Illinois, 1308 West Main Street, Urbana, IL. 61801

an ϵ-net [3], for which the sample size given in [1] is indeed the appropriate one; however, it does not allow us to dispense with this factor.

It should also be noted that Lemma 4.13 continues to follow from (1) with $\mathcal{F} = \mathcal{D}'$ and $\epsilon = 16\sqrt{\frac{\log n}{n}}$, since $d(\mathcal{D}') \leq 30$ and c_5 can be chosen as 16 plus the constant in Lemma 4.9. $\square$

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References

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