Proportionally Fair Distributed Resource Allocation in Multi-Band Wireless Systems

I-Hong Hou and Piyush Gupta

Abstract—A challenging problem in multi-band multi-cell self-organized wireless systems, such as femtocells/picocells in cellular networks, multi-channel Wi-Fi networks, and more recent wireless networks over TV white spaces, is of distributed resource allocation. This in general involves four components: channel selection, client association, channel access, and client scheduling. In this paper, we present a unified framework for jointly addressing the four components with the global system objective of maximizing the clients throughput in a proportionally fair manner. Our formulation allows a natural dissociation of the problem into two sub-parts. We show that the first part, involving channel access and client scheduling, is convex and derive a distributed adaptation procedure for achieving a Pareto-optimal solution. For the second part, involving channel selection and client association, we develop a Gibbs-sampler based approach for local adaptation to achieve the global objective, as well as derive fast greedy algorithms from it that achieve good solutions often.

Index Terms—Wireless resource allocation; multi-band spectrum access; femtocells; IEEE 802.11 systems; TV white spaces.

I. INTRODUCTION

Many of the existing and evolving wireless systems operate over spectrum that spans multiple bands. These bands may be contiguous, as in OFDM-based systems, such as current IEEE 802.11-based WLANs (a.k.a. Wi-Fi networks) and evolving fourth-generation LTE cellular wireless systems; or they may be spread far apart, as in multi-channel 802.11 systems and in recently proposed wireless broadband networks over TV white spaces (discussed in Section III). A common issue in these multi-band systems is how to perform resource allocation among different clients, possibly being served by different access points (APs). This needs to be done so as to efficiently utilize wireless resources—spectrum, transmission opportunities and power—while being fair to different clients. Furthermore, unlike traditional enterprise Wi-Fi networks and cellular wireless networks, where the placement of APs and their operating bands are arrived at after careful capacity/coverage planning, more and more of the evolving wireless systems are going to be self-organized networks. There is extensive literature on completely self-organized wireless networks, also referred to as *ad hoc*

I-Hong Hou is with Department of ECE, Texas A&M University, College Station, TX 77843, email:ihou@tamu.edu

Piyush Gupta is with Bell Labs, Alcatel-Lucent, 600 Mountain Ave., 2A-427, Murray Hill, NJ 07974, email: pgupta@research.bell-labs.com

networks [1], which are often based on 802.11. Even the emerging 4G cellular wireless networks, such as those based on LTE, are going to have a significant deployment of self-organized subsystems: namely, femtocells and picocells [2], [3]. These self-organized (sub)-systems will require that resource allocation is performed dynamically in a distributed manner and with minimal coordination between different APs and/or clients.

In this paper, we consider the problem of joint resource allocation across different APs and their clients so as to achieve a global objective of maximizing the system throughput while being fair to users. We focus on achieving proportional fairness, which has become essentially standard across current 3G cellular systems, as well as in emerging 4G systems based on LTE and WiMAX. Thus, the system objective will be to allocate wireless resources, spectrum and transmission opportunities, so as to maximize the (weighted) sum of the log of throughputs of different clients, which is known to achieve (weighted) proportional fairness.

The combined resource allocation in a multi-band multi-cell wireless system involves four components: Channel Selection, Client Association, Channel Access, and Client Scheduling. The first component, Channel Selection, decides on how different APs share different bands of the spectrum available to the wireless system. The second component, Client Association, allows a client to decide on an AP to associate within its neighborhood that is likely to provide the "best" performance. Once an AP has chosen a channel/band to operate in and a bunch of clients have associated with it, the third component, Channel Access, decides when it should access the channel so as to serve its clients while being fair to other access points in its neighborhood operating in the same channel. The final component, Client Scheduling, decides which of its clients an AP should serve whenever it successfully accesses the channel.

Our approach addresses the four components in a unified framework, where the solutions to different components are arrived at through separation of time scales of adaptation. More specifically, our formulation allows for the optimization problem of maximizing the clients throughput with weighted proportional fairness to naturally dissociate into two sub-parts, which are adapted at different time scales. Assuming that the channel selection and client association have been performed, we show that the part involving channel access and client scheduling becomes convex, which is also amenable to a distributed

adaptation for achieving Pareto-optimal weighed proportional fairness. At a slower time scale, we adapt the channel selection and client association to varying demands and interference. This part is a non-convex problem in general, and thus, difficult to solve for a globally optimal solution. We develop a Gibbs-sampler based approach to perform local adaptation while improving the global system objective. The adaptation is randomized, and if done slowly enough, can achieve a globally optimal solution. In practice, however, that may not be always feasible; hence, we derive greedy heuristics from it for channel selection and client association, which though not globally optimal, provide fast and good distributed solutions with limited exchange of information, as the simulation results indicate. Simulations also illustrate that our resource-allocation policies can achieve significantly better performance than state-of-the-art techniques, such as those proposed in [4] and [5].

The paper is organized as follows. Section II gives an overview of related work. Section III describes the multi-cell multi-band wireless system model and the joint resource allocation problem that we study. Section IV gives an overview of our approach and discusses the separation of problems. The details of our approach and its desirable properties, including convergence to Pareto-optimal proportionally-fair allocation, are given in Section V and Section VI. The results of simulations are provided in Section VII. Concluding remarks are discussed in Section VIII.

II. RELATED WORK

There is extensive literature addressing one or a subset of the aforementioned four components of wireless resource allocation in the context of different wireless systems—Wi-Fi networks, 3G/4G cellular networks, and more recent, wireless broadband systems over TV white spaces. Due to space limitation, we only discuss a small sample of the results in each of these areas.

Distributed resource allocation has been widely studied for IEEE 802.11-based systems. A number of CSMA-based random-access approaches have been developed to provide differentiated services to clients [6]–[11]. Proportional fairness in multi-contention neighborhood has been studied in [12], [13]. Specifically, [13] has established that Pareto-optimal weighted proportional fairness can be achieved in a distributed manner with minimal exchange of information among contending clients. Our approach for channel access is motivated by [13]; we, however, consider a more general framework that jointly addresses all the four components. Random-access policies that achieve the entire throughput region have been studied under both the protocol model [14] and the physical model [15]. These studies focus on adapting the access probabilities of each link but do not incorporate the other control components: channel allocation and client association. Multi-channel MACs for Wi-Fi networks have been proposed in [16], [17]. Approximation algorithms

for client association control to achieve proportional fairness have been developed in [5], [18], but they assume that APs do not interfere with each other through a pre-assignment of orthogonal channels. A framework based on Markov Approximation has been proposed for combinatorial network optimization problems [19]. However, this work requires each node in the system to obtain system-wide performance. Further, it only provides approximation solutions. The work closest to this paper is [4] where the authors have developed Gibbs-sampler based distributed algorithms for channel selection and client association. Their approach, however, considers different objectives for the two components, neither of which ensures proportional fairness. On the other hand, [20] and [21] have considered the joint optimization for channel selection, user association, and power control and have proposed algorithms that achieve minimum system-wide potential delay, without considering the possibility of time-sharing or random access.

For cellular wireless data systems, a number of centralized approaches for single-cell scheduling to achieve proportional fairness have been developed [22]–[24]. More recently, several inter-cell interference coordination (ICIC) techniques have been proposed for interference mitigation in LTE-based 4G cellular networks. Specifically, [25], [26] have developed distributed algorithms for dynamic fractional frequency reuse among interfering macro cells based on limited exchange of interference information over dedicated control links. Given the non-convexity of the problem, these algorithms aim to achieve a local optimum of the weighted sum of the log of user throughputs, which as mentioned earlier provides proportional fairness, through local estimates of the appropriate gradients. However, in self-organized subsystems of such networks (such as those made of femtocells and picocells), explicit exchange of information may not be feasible [27], and thus, these approaches may not be directly applicable. [28] has considered the problem of jointly achieving proportional fairness and energy efficiency in LTE systems. It considers a different model where interference only causes the data rate to decrease and does not result in packet collisions. [29] also uses the Gibbs sampler approach to address the problem of utility maximization in general wireless networks. However, its approach may result in excessive complexity and long convergence times. In contrast, by leveraging the knowledge of the optimal solution to some of the control components, our policy has much smaller complexity and faster convergence times.

Research on resource allocation in wireless networks operating over fragmented TV white spaces is in a nascent stage. [30] considers a single AP serving multiple clients at the same rate. For this scenario, it addresses three issues: how the AP chooses a suitable band, how a new client detects the AP's operating band, and how disruptions due to temporal variations, such as caused by wireless microphones, are handled. Some of the limitations of this approach are overcome in [31] by considering

a multi-rate multi-radio architecture. It develops a joint strategy for white space selection and client assignment to one of the radios, as well as designs an extension to CSMA to achieve proportional fairness—all for a single-AP scenario. This paper generalizes the setup by considering a system of multiple APs and jointly addressing the four components of resource allocation to achieve the global system objective of maximizing clients throughput in a proportionally fair manner.

III. SYSTEM MODEL

We consider a system with a finite number of access points (APs) and clients that can operate in a number of channels. We denote the set of APs by $\mathbb{N}$, the set of clients by $\mathbb{I}$, and the set of channels by $\mathbb{C}$. Each client i is associated with an AP and is served by that AP. APs are connected to a backhaul network and can communicate with each other over this network. In Section VI, we will show that, each AP only needs to communicate a small amount of data with APs that are physically close to it under our approach. Hence, our approach only causes a small overhead on the backhaul network.

In this paper, we use i, j, k, l to denote clients, and n, m, o to denote APs. Many variables can be defined from both a client's perspective and an AP's perspective. In this case, we use subscript when the variable is defined from a client's perspective, and use superscript when the variable is defined from an AP's perspective. For example, x_i is the value of x from client i's perspective, while x^n is the value of x from AP n's perspective.

We denote the AP that client i is associated with by $n_i \in \mathbb{N}_i$, where $\mathbb{N}_i$ is the set of APs that client i can be associated with, subject to both privacy settings of APs and the geographical location of the client. Each AP n is equipped with u^n radios that can operate in different channels simultaneously and each client i is equipped with only one radio. When an AP n has more than one radios, i.e, $u^n > 1$, we can simplify the model by assuming that there are u^n APs, each with one radio, that are placed at the same place as AP n. Each of these u^n APs corresponds to one radio of AP n. This procedure allows us to only consider the model in which each AP has one radio and simplifies notations. Our joint resource-allocation approach will ensure that the co-located APs operate on different bands whenever necessary to optimize the system objective. Hence, throughout the rest of the paper, we assume that each AP only has one radio and operates in one channel unless otherwise specified. The channel that an AP n is operating in is denoted by c^n. APs can switch the channels that they operate in, although such switches can only be done infrequently due to the large overheads. When an AP switches channels, all its clients also switch channels accordingly. Fig. 1 shows an example of such a system with 4 APs, 13 clients, and 2 channels. In the figure, each client is connected to the AP that it is associated with. We use a solid line to indicate that the AP is operating in channel 1, and a dashed line to indicate that the AP is operating in channel 2.
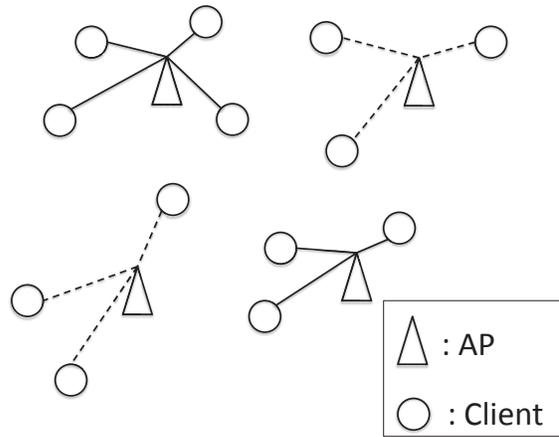

Fig. 1: An example of a system with 4 APs, 13 clients, and 2 channels.

We focus on a server-centric scheme where each AP schedules all transmissions between itself and all clients that are associated with it. This scheme can be naturally applied when all transmissions are downlink ones. Even when there are uplink transmissions, it is still applicable to a wide varieties of wireless systems that include LTE, WiMax, and IEEE 802.11 PCF. Later, we will also discuss a fully distributed scheme where clients contend for service from APs, such as in 802.11 DCF.

We assume that time is slotted, with the duration of a time slot equals the time needed for a transmission. If an AP n makes a successful transmission toward client i in channel c, client i receives data at rate of $B_{i,n,c}$ in this slot. Since the characteristics of different channels may be different, $B_{i,n,c}$ depends on c. We set $B_{i,n,c} = 0$ for all $n \notin \mathbb{N}_i$.

Given the emphasis on self-organized networks, we assume that the APs are not synchronized and may interfere with each other. We consider the interference relations using the protocol model [32]. When an AP n operates in channel c, it may be interfered by a subset $\mathbb{M}^{n,c}$ of APs, if these APs were to also operate in channel c. We let $n \in \mathbb{M}^{n,c}$ for notational simplicity. When the AP n schedules a transmission between itself and one of its clients over channel c, the transmission is successful if n is the only AP among $\mathbb{M}^{n,c}$ that transmits in channel c during the time of transmission; otherwise, the transmission suffers from a collision and fails. We assume that the interference relations are symmetric, i.e., $m \in \mathbb{M}^{n,c}$ if and only if $n \in \mathbb{M}^{m,c}$. Note that, since the propagation characteristics of different channels may be different, especially in the case of TV white space access, the subset $\mathbb{M}^{n,c}$ depends on the channel c. This dependency further distinguishes our work from most existing works on multi-channel access where interference relations are assumed to be identical for all channels. To further simplify notation, we also define $\overline{\mathbb{M}}^{n,c} := \{m | m \in \mathbb{M}^{n,c}, c^m = c\}$, which is the set of APs that *actually* interfere with n when it operates in channel c. The main difference between $\mathbb{M}^{n,c}$ and $\overline{\mathbb{M}}^{n,c}$ is

that the former describes the physical constraints of the system and is independent of the actual channel that each AP operates in, while the latter depends on the channel that each AP operates in. We also define, for each client i, the set of APs that may interfere with an AP that client i can be associated with as

$$\mathbb{M}_i := \{m \in \mathbb{N} | \exists c \in \mathbb{C}, n \in \mathbb{N}_i \text{ s.t. } m \in \mathbb{M}^{n,c}\}.$$

In the server-centric scheme, each AP is in charge of scheduling transmissions for its clients. When AP n accesses the channel, it schedules the transmission for client i, where $n_i = n$, with probability $\phi_{i,n}$, $\phi_{i,n} \geq 0$ and $\sum_{i:n_i=n} \phi_{i,n} = 1$.

Since APs are not coordinated, we assume that they access the channel by random access. Each AP n chooses a random access probability, p^n. In each slot, AP n accesses the channel c^n with probability p^n. The transmission is successful if n is the only AP in $\mathbb{M}^{n,c^n}$ that transmits over channel c^n. Thus, the probability that AP n successfully accesses the channel in a time slot can be expressed as

$$p^n \prod_{m \in \overline{\mathbb{M}}^{n,c^n}, m \neq n} (1 - p^m)$$
$$= \frac{p^n}{1 - p^n} \prod_{m \in \overline{\mathbb{M}}^{n,c^n}} (1 - p^m). \tag{1}$$

Assume that client i is associated with AP n and operate in channel c, i.e., $n_i = n$ and $c^n = c$. Since the data rate of client i when it is served is $B_{i,n,c}$, and the probability that the AP n makes a successful transmission is as in Eq. (1), its throughput per time slot is,

$$r_i := B_{i,n,c} \phi_{i,n} \frac{p^n}{1 - p^n} \prod_{m \in \overline{\mathbb{M}}^{n,c}, c^m = c} (1 - p^m). \tag{2}$$

We now discuss an analog model for a fully distributed scheme where clients contend for the service from APs, which is applicable to completely distributed scenarios, such as those based on 802.11 DCF. In this scheme, each client i contends for the channel by accessing it with probability p_i in each time slot. Two clients, i and j, interfere with each other if their associated APs interfere with each other, that is, $c^{n_i} = c^{n_j}$ and $n_i \in \mathbb{M}^{n_j, c^{n_j}}$. Client i successfully accesses the channel if none of the other clients that can interfere with it accesses the channel simultaneously. Thus, the long-term throughput per time slot for client i is, assuming $n_i = n$ and $c^n = c$,

$$r_i^D := B_{i,n,c} \frac{p_i}{1 - p_i} \prod_{j:n_j \in \overline{\mathbb{M}}^{n,c}} (1 - p_j).$$

Finally, we assume that each client is associated with a positive weight $w_i > 0$. We also denote the total weights of clients associated with AP n by w^n, i.e., $w^n := \sum_{i:n_i=n} w_i$. The goal is to achieve weighted proportional fairness among clients, that is, to maximize $\sum_{i \in \mathbb{I}} w_i \log r_i$. In the fully distributed scheme, the goal is to maximize $\sum_{i \in \mathbb{I}} w_i \log r_i^D$.

As a final remark, in a dynamic system, the APs and clients may update their values of different parameters, such as n_i, c^n, p^n, etc, with time. However, for simplicity, we omit their dependence on time from the notation. Throughput this paper, these variables should be interpreted as their current values.

IV. SOLUTION OVERVIEW AND TIME-SCALE SEPARATION

We now give an overview of our approach to achieve weighted proportional fairness, which consists of decomposing the problem into four components and solving them. The solution to the fully distributed scheme is very similar to that to the server-centric scheme. Thus, we will focus on the server-centric scheme and only report the key differences of the fully distributed scheme.

By Eq. (2), we can formulate the problem of achieving weighted proportional fairness as the following optimization problem:

$$\max \sum_i w_i \log r_i$$
$$= \sum_i w_i [\log B_{i,n_i,c^{n_i}} + \log \phi_{i,n_i} + \log \frac{p^{n_i}}{1 - p^{n_i}}$$
$$+ \log \prod_{m \in \overline{\mathbb{M}}^{n_i, c^{n_i}}} (1 - p^m)],$$
$$\text{s.t. } c^n \in \mathbb{C}, \text{ for all } n;$$
$$n_i \in \mathbb{N}_i, \text{ for all } i;$$
$$0 \leq p^n \leq 1, \text{ for all } n;$$
$$\phi_{i,n_i} \geq 0, \text{ for all } i;$$
$$\sum_{i:n_i=n} \phi_{i,n} = 1, \text{ for all } n.$$

Based on this formulation, the problem of achieving weighted proportional fairness consists of four important components, in increasing order of time scales: First, whenever the AP accesses the channel, it needs to schedule one client for service. That is, the AP has to decide the values of $\phi_{i,n}$. Second, in each time slot, the AP has to decide whether it should access the channel, which consists of determining the values of p^n. Third, each client needs to decide which AP it should be associated with, i.e., deciding n_i. Finally, each AP n needs to choose a channel, c^n, to operate in. We denote the four components as *Scheduling Problem*, *Channel Access Problem*, *Client Association Problem*, and *Channel Selection Problem*, respectively. Weighted proportional fairness is achieved by jointly solving the four components.

The problem of achieving weighted proportional fairness for the fully distributed scheme can be formulated

similarly as follows:

$$\max \sum_i w_i \log r_i^D$$
$$= \sum_i w_i [\log B_{i,n_i,c^{n_i}} + \log \frac{p_i}{1-p_i}$$
$$+ \log \prod_{j:n_j \in \overline{\mathbb{M}}^{n_i,c^{n_i}}} (1-p_j)],$$
$$\text{s.t. } c^n \in \mathbb{C}, \text{ for all } n;$$
$$n_i \in \mathbb{N}_i, \text{ for all } i;$$
$$0 \le p_i \le 1, \text{ for all } i.$$

This involves three components: the Channel Access Problem, which chooses p_i for clients, the Client Association Problem, and the Channel Selection Problem.

Since the overhead for a client to change the AP it is associated with and for an AP to change the channel it operates in are high, solutions to the Client Association Problem and the Channel Selection Problem are updated at a much slower time scale compared to solutions to the Scheduling Problem and the Channel Access Problem. Based on this timescale separation, we first study the solutions to the Scheduling Problem and the Channel Access Problem, given fixed solutions to the Client Association Problem and the Channel Selection Problem. We will show that, given solutions to the Client Association Problem and the Channel Selection Problem, there are closed-form expressions for the optimal solutions to the Scheduling Problem and the Channel Access Problem. We then study the solutions to the Client Association Problem and the Channel Selection Problem, under the knowledge of how solutions to the Scheduling Problem and the Channel Access Problem react. Whenever the solutions to the Client Association Problem and the Channel Selection Problem are updated, the solutions to the Scheduling Problems and the Channel Access Problems are also updated according to the optimal closed-form expressions. Thus, solutions to the Client Association Problem and the Channel Selection Problem are indeed joint solutions to all the four components, and their optimal solutions achieve Pareto-optimal weighted proportional fairness. In addition to solving the four components, we will show that the solutions naturally turn into distributed algorithms where each client/AP makes decisions based on local knowledge.

V. THE SCHEDULING PROBLEM AND THE CHANNEL ACCESS PROBLEM

In this section, we assume that solutions to the Client Association Problem and the Channel Selection Problem, i.e., n_i and c^n, are fixed.

Since n_i and c^n are fixed, values of $B_{i,n_i,c^{n_i}}$ are con-

stant. The optimization problem can be rewritten as

$$\text{Max} \sum_{i \in \mathbb{I}} w_i [\log \phi_{i,n_i} + \log \frac{p^{n_i}}{1-p^{n_i}}$$
$$+ \log \prod_{m \in \overline{\mathbb{M}}^{n_i,c^{n_i}}} (1-p^m)]$$
$$= \sum_{i \in \mathbb{I}} w_i \log \phi_{i,n_i} + \sum_{n \in \mathbb{N}} [w^n \log p^n$$
$$+ (\sum_{m \in \overline{\mathbb{M}}^{n,c^n}} w^m - w^n) \log(1-p^n)],$$
$$\text{s.t. } 0 \le p^n \le 1, \text{ for all } n; \quad \phi_{i,n_i} \ge 0, \text{ for all } i;$$
$$\sum_{i:n_i=n} \phi_{i,n} = 1, \text{ for all } n,$$

where $w^n = \sum_{i:n_i=n} w_i$, as defined in Section III. We also define $z^n := \sum_{m \in \overline{\mathbb{M}}^{n,c^n}} w^m$ to be the total weight of the clients that are associated with APs interfering with n, including itself.

This formulation naturally decomposes the optimization problem into two independent parts: maximizing $\sum_i w_i \log \phi_{i,n_i}$ over $\phi_{i,n}$, which is the Scheduling Problem, and maximizing $\sum_n [w^n \log p^n + (z^n - w^n) \log(1-p^n)]$ over p^n, which is the Channel Access Problem. Thus, we can solve these two problems independently.

We first solve the Scheduling Problem in the following.

Theorem 1: Given n_i, c^n, and p^n, for all i and n, $\sum_i w_i \log r_i$ is maximized by $\phi_{i,n_i} \equiv w_i/w^{n_i}$.

Proof: We have $\frac{\partial}{\partial \phi_{i,n_i}} (\sum_{j \in \mathbb{I}} w_j \log r_j) = \frac{w_i}{\phi_{i,n_i}}$. Since $\sum_i w_i \log r_i$ is concave in $[\phi_{i,n}]$ with the condition $\sum_{i:n_i=n} \phi_{i,n} = 1$, we have that $\frac{w_i}{\phi_{i,n}} = \frac{w_j}{\phi_{j,n}}$, for all i,j such that $n_i = n(j) = n$, at the optimal point. By setting $\phi_{i,n_i} \equiv w_i/w^{n_i}$, the aforementioned criterion is satisfied. The conditions $\phi_{i,n_i} \ge 0$, for all i, and $\sum_{i:n_i=n} \phi_{i,n} = 1$, for all n, are also satisfied. Thus, the Scheduling Problem is solved by setting $\phi_{i,n_i} \equiv w_i/w^{n_i}$. ∎

We address the Channel Access Problem next. The following is a direct consequence of Theorem 1 in [13].

Theorem 2: Given n_i, c^n, and $\phi_{i,n}$, for all i and n, $\sum_i w_i \log r_i$ is maximized by $p^n \equiv w^n/z^n$.

In summary, when the solutions to the Client Association Problem and the Channel Selection Problem, i.e., n_i and c^n, are fixed, the AP n should access the channel with probability $p_n = w^n/z^n$ in each time slot and should schedule the transmission for its client i with probability $\phi_{i,n} = w_i/w^n$ whenever it accesses the channel. In addition to achieving the optimal solution to both the Scheduling Problem and the Channel Access Problem, this solution only requires n to know the local information of w^m and c^m for all AP m that it interferes with. Thus, this solution can be easily implemented in a distributed manner.

We now discuss the fully distributed scheme. With the solutions to the Client Association Problem and the Channel Selection Problem fixed, the problem of achieving proportional fairness is equivalent to maximizing $\sum_i w_i \log p_i + (z^{n_i} - w_i) \log(1-p_i)$. A result similar to

Theorem 2 shows that the Channel Access Problem is optimally solved by choosing $p_i = w_i/z^{n_i}$.

VI. THE CLIENT ASSOCIATION PROBLEM AND THE CHANNEL SELECTION PROBLEM

We now propose a distributed algorithm that solves the Client Association Problem and the Channel Selection Problem based on the knowledge of the optimal solutions to the Scheduling Problem and the Channel Access Problem. These two problems are non-convex and a locally optimal solution to the two problems may not be globally optimum. Thus, common techniques for solving convex problems are not suitable for these problems. Instead, the proposed algorithm uses a simulated annealing technique that is based on the Gibbs Sampler, see, e.g. [33], which is proven to converge to the global optimum point almost surely. At the end of Section VI-B, we also derive a greedy heuristic that is easier to implement and converges faster. We first introduce the concept of the Gibbs Sampler.

A. The Gibbs Sampler

Consider a system that contains a set of V entities. Each entity $v \in V$ can be in a state within a *state space* $\Lambda = \{\lambda\}$, and we denote by $X_v \in \Lambda$ the state of entity v. Define the *configuration* of the system as the vector $\psi := [X_v, v \in V]$ consisting of the states of all entities in the system. Let Ψ be the set of all possible configurations.

Define a *utility function* on the configuration of the system $U : \Psi \mapsto R$. The Gibbs sampler is a distributed approach that aims to find the configuration that maximizes the utility function.

Using the Gibbs sampler, each entity updates its state over time. The updates can be taken in either synchronously, e.g., each entity updates in a round-robin fashion, or asynchronously, e.g., each entity uses an independent exponential timer. Suppose an entity v_t is to make an update at time t. Assume that the configuration of the system at time t is ψ_t and denote by $\psi_t(X_{v_t} = \lambda)$ as the configuration where the entity v_t is in state λ and all entities other than v_t are in the same states as they are in ψ_t. The entity v_t then chooses to be in state λ_t, and therefore $\psi_{t+1} = \psi_t(X_{v_t} = \lambda_t)$, with probability

$$\pi_t(X_{v_t} = \lambda_t) = \frac{e^{U(\psi_t(X_{v_t}=\lambda_t))/T_t}}{\sum_{\lambda \in \Lambda} e^{U(\psi_t(X_{v_t}=\lambda))/T_t}}, \qquad (3)$$

where T_t is referred as the *temperature* of the system and is decreasing in t. If T_t is chosen so that $T_t \to 0$ and $T_t \log t \to \infty$ as t goes to infinity. More discussion on this can be found in Chapter 7 [33].

B. A Distributed Protocol

We now apply the Gibbs Sampler for solving the Client Association Problem and the Channel Selection Problem. We call a joint solution to both the Client Association Problem and the Channel Selection Problem as a configuration of the system. A configuration is thus fully specified

by the AP each client is associated with, and the channel each AP operates in. As we aim to achieve proportional fairness among the system, the utility of the system under configuration ψ_t, $U(\psi_t)$, is defined to be as the value of $\sum_i w_i \log r_i$ when APs and clients choose their channels to operate in and APs to be associated with according to ψ_t, and apply the optimal solution to the Scheduling Problem and the Channel Access Problem under ψ_t. We then have

$$U(\psi_t) = \sum_{i \in \mathbb{I}} w_i [\log B_{i,n_i,c(n_i)} + \log \frac{w_i}{w^{n_i}}] + \sum_{n \in \mathbb{N}} [w^n \log \frac{w^n}{z^n} + (z^n - w^n) \log \frac{z^n - w^n}{z^n}]. \tag{4}$$

Finding the joint solution that achieves Pareto-optimal proportional fairness is equivalent to finding the configuration ψ that maximizes $U(\psi)$.

At each time t, either a client or an AP is selected in a random or round-robin fashion. The selected client, or AP, then changes the AP it is associated with, or the channel it operates in, randomly in a manner described in the following paragraphs, while all other clients and APs make no changes. The solutions to the Scheduling Problem and the Channel Access Problem are then updated according to the new configuration.

We now discuss how the selected client, or AP, changes the AP it is associated with, or the channel it operates in, respectively. Let $\psi_t(n_i = n)$ be the configuration where client i is associated with AP n, and the remaining of the system is the same as in configuration ψ_t. We can define $\psi_t(c^n = c)$ for AP n similarly. If client i is selected at time t, it changes the AP it is associated with to n with probability $e^{U(\psi_t(n_i=n))/T(t)} / \sum_m e^{U(\psi_t(n_i=m))/T(t)}$, where $T(t)$ is a positive decreasing function. On the other hand, if AP n is selected at time t, it changes the channel it operates in to c with probability $e^{U(\psi_t(c^n=c))/T(t)} / \sum_d e^{U(\psi_t(c^n=d))/T(t)}$.

It remains to compute the values of $U(\psi_t(n_i = n))$ for client i and $U(\psi_t(c^n = c))$ for AP n. We first discuss how to compute $U(\psi_t(n_i = n))$. Let $w^n_{-i} := \sum_{j:n_j=n, j\neq i} w_j$ be the total weight of clients, excluding i, associated with AP n. Let $z^n_{-i} := \sum_{m \in \overline{\mathbb{M}}^{n,c^n}} w^m_{-i}$. Define

$$U^0_i(\psi_t)$$
$$= \sum_{j \in \mathbb{I}, j \neq i} w_j [\log B_{j,n_j,c^{n_j}} + \log \frac{w_j}{w^{n_j}_{-i}}]$$
$$+ \sum_{n \in \mathbb{N}} [w^n_{-i} \log \frac{w^n_{-i}}{z^n_{-i}} + (z^n_{-i} - w^n_{-i}) \log \frac{z^n_{-i} - w^n_{-i}}{z^n_{-i}}],$$

which can be thought of as the utility of the system as if the weight of client i were zero. We then define $\Delta U^n_i(\psi_t) := U(\psi_t(n_i = n)) - U^0_i$. Since in the configuration $\psi_t(n_i = n)$, $w^m = w^m_{-i}$ for all $m \neq n$; $w^n = w^n_{-i} + w_i$; $z^m = z^m_{-i} + w_i$ if $m \in \overline{\mathbb{M}}^{n,c^n}$, and $m \neq n$; and $z^m = z^m_{-i}$,

otherwise, we have

$$\Delta U_i^n(\psi_t)$$

$$= w_i[\log B_{i,n,c^n} + \log \frac{w_i}{w^n}] + \sum_{j:n_j=n} w_j \log \frac{w_{-i}^n}{w_{-i}^n + w_i}$$

$$+ w_i \log \frac{w^n}{z^n} + w_{-i}^n \log \frac{z_{-i}^n(w_{-i}^n + w_i)}{w_{-i}^n(z_{-i}^n + w_i)}$$

$$+ (z_{-i}^n - w_{-i}^n) \log \frac{z_{-i}^n}{z_{-i}^n + w_i}$$

$$+ \sum_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} [w_{-i}^m \log \frac{z_{-i}^m}{z_{-i}^m + w_i}$$

$$+ (z_{-i}^m - w_{-i}^m) \log \frac{(z_{-i}^m - w_{-i}^m + w_i)z_{-i}^m}{(z_{-i}^m + w_i)(z_{-i}^m - w_{-i}^m)}$$

$$+ w_i \log \frac{z^m - w^m}{z^m}]$$

$$= w_i[\log \frac{B_{i,n,c^n}w_i}{z^n} + \sum_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} \log \frac{z^m - w^m}{z^m}]$$

$$+ \sum_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} [\log(1 + \frac{w_i}{z_{-i}^m - w_{-i}^m})^{z_{-i}^m - w_{-i}^m}$$

$$- \log(1 + \frac{w_i}{z_{-i}^m})^{z_{-i}^m}] + \log(1 - \frac{w_i}{z_{-i}^n + w_i})^{z_{-i}^n}$$

$$\approx w_i\left[\log\left(\frac{B_{i,n,c^n}w_i}{z^n} \prod_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} \frac{z^m - w^m}{z^m}\right)\right]$$

$$+ \alpha,$$

where α is a constant. Since $(1 + \frac{w_i}{A})^A \approx e^{w_i}$ and $(1 - \frac{w_i}{A})^A \approx e^{-w_i}$ for all $A >> w_i$, the last approximation holds when $z_{-i}^m >> w_i$, which is true in a dense network where the weights of all clients are within the same order.

Suppose a client i is selected to change its state at time t, at which time the configuration of the system is ψ_t. The probability that i chooses AP n to be associated with is

$$e^{[U_i^0(\psi_t)+\Delta U_i^n(\psi_t)]/T(t)} / \sum_m e^{[U_i^0(\psi_t)+\Delta U_i^m(\psi_t)]/T(t)}$$

$$\approx \frac{1}{\gamma}\left(\frac{B_{i,n,c^n}w_i}{z^n} \prod_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} \frac{z^m - w^m}{z^m}\right)^{w_i/T(t)}, \quad (5)$$

where γ is the normalizer.

To compute the probability of choosing AP n to be associated with, client i only needs the values of B_{i,n,c^n} for all $n \in \mathbb{N}_i$, w^m and z^m for all $m \in \mathbb{M}_i$. Thus, this probability can be computed by client i using its local information. We also note that this probability has the following properties: First, it increases with B_{i,n,c^n}, meaning that client i tends to choose the AP that has higher data rate; Second, it decreases with z^n, which is the total weights of clients that interfere with n; Finally, it increases with $\prod_{m\in\overline{\mathbb{M}}^{n,c^n},m\neq n} \frac{z^m-w^m}{z^m}$, which is the probability that none of the APs that interfere with n access the channel in a time slot. Thus, this probability jointly considers the three important factors for the Client

Association Problem: data rate, interference, and channel congestion.

Next we discuss the computation of the probability that an AP n should choose channel c to operate in, if it is selected. Let $z_{-n}^m := \sum_{o\in\overline{\mathbb{M}}^{m,c^m},o\neq n} w^o - w^m$. Let $U_n^0(\psi_t)$ be the utility of the system under configuration ψ_t, if the weights of all its clients were zero. That is,

$$U_n^0(\psi_t)$$

$$= \sum_{j\in\mathbb{I},n_j\neq n} w_j[\log B_{j,n_j,c^{n_j}} + \log \frac{w_j}{w^{n(j)}}]$$

$$+ \sum_{m\in\mathbb{N},m\neq n} [w^m \log \frac{w^m}{z_{-n}^m} + (z_{-n}^m - w^m) \log \frac{z_{-n}^m - w^m}{z_{-n}^m}].$$

We then define $\Delta U_n^c(\psi_t) := U(\psi_t(c^n = c)) - U_n^0$. Since in the configuration $\psi_t(c^n = c)$, $z^m = z_{-n}^m + w^n$ if $m \in \overline{\mathbb{M}}^{n,c}$, and $m \neq n$; and $z^m = z_{-n}^m$, otherwise, we have

$$\Delta U_n^c(\psi_t)$$

$$= \sum_{i:n_i=n} w_i[\log B_{i,n,c} + \log \frac{w_i}{w^n}] + w^n \log \frac{w^n}{z^n}$$

$$+ (z^n - w^n) \log \frac{z^n - w^n}{z^n}$$

$$+ \sum_{m\in\overline{\mathbb{M}}^{n,c},m\neq n} [w^m \log \frac{z_{-n}^m}{z_{-n}^m + w^n}$$

$$+ (z_{-n}^m - w^m) \log \frac{(z_{-n}^m - w^m + w^n)z_{-n}^m}{(z_{-n}^m + w^n)(z_{-n}^m - w^m)}$$

$$+ w^n \log \frac{z_{-n}^m - w^m + w^n}{z_{-n}^m + w^n}].$$

When an AP n is selected by the Gibbs sampler at time t, it changes the channel it operates in randomly, with the probability of changing to channel c proportional to $e^{U(\psi_t(c^n=c))/T(t)} = e^{[U_n^0(\psi_t)+\Delta U_n^c(\psi_t)]/T(t)} \propto e^{\Delta U_n^c(\psi_t)/T(t)}$. We note that, to compute $\Delta U_n^c(\psi_t)$, AP n only needs the values of $B_{i,n,c}$, w^i, for each client i that is associated with n, and z_{-n}^m, w^m for all $m \in \cup_c\mathbb{M}^{n,c}$. Thus, $\Delta U_n^c(\psi_t)$ can also be computed using only local information.

From the above discussion, a distributed protocol based on the Gibbs sampler, referred to as DP, can be designed (see Algorithm 1). In DP, all clients and APs in the system only need to exchange information within their local areas, as they only need local information to compute the probability of choosing an AP to be associated with or a channel to operate in. Thus, DP is easily scalable. Further, DP achieves Pareto-optimal proportional fairness as $t \to \infty$ almost surely by Theorem **??**.

In addition to DP, we can also consider a greedy policy (Greedy) that is easier to implement (see Algorithm 2). Greedy works similar to DP, except that when a client i, or an AP n, is selected by the sampler, it chooses the AP that maximizes $U(\psi_t(n_i = n))$ to be associated with, or the channel that maximizes $U(\psi_t(c^n = c))$ to operate in, respectively. It is essentially a coordinate ascent approach. As the number of configurations is finite, it is guaranteed to converge to a local optimum. In addition to a simpler implementation, Greedy is also consistent with the selfish

behavior of clients. Each client i chooses the AP n that maximizes $\frac{B_{i,n,c^n}}{z^n} \prod_{o \in \overline{\mathbb{M}}^{n,c^n}, o \neq n} \frac{z^o}{z^o + w^o}$, which is indeed the value of r_i when i is associated with n. Thus, in Greedy, every client always chooses to associate with the AP that offers the highest throughput.

A similar protocol can also be designed for the fully distributed scheme where clients, instead of APs, contend for channel access. The protocol also uses the Gibbs sampler as in DP. Define z_{-i}^n, z_{-n}^m, $U_i^0(\psi_t)$, $\Delta U_i^n(\psi_t)$, $U_n^0(\psi_t)$, and $\Delta U_n^c(\psi_t)$ similar to those in the server-centric scheme. We can derive that, for the distributed protocol,

$$\Delta U_i^n(\psi_t)$$
$$= w_i \log(B_{i,n,c^n} \frac{w_i}{z_{-i}^n + w_i} \prod_{j:n_j \in \overline{\mathbb{M}}^{n,c^n}, j \neq i} \frac{z_{-i}^{n_j} - w_j + w_i}{z_{-i}^{n_j} + w_i})$$
$$- z_{-i}^n \log \frac{z_{-i}^n + w_i}{z_{-i}^n},$$

and

$$\Delta U_n^c(\psi_t)$$
$$= \sum_{i:n_i=n} w_i \log B_{i,n,c} \frac{w_i}{z^n} + \sum_{i:n_i=n} (z^n - w_i) \log \frac{z^n - w_i}{z^n}$$
$$+ \sum_{m \in \overline{\mathbb{M}}^{n,c^n}, m \neq n} w^m \log \frac{z_{-n}^m}{z_{-n}^m + w^n}.$$

We close this section by discussing some implementation issues. The Gibbs sampler requires clients and APs to exchange information locally. Many steps in Algorithm 1 involve broadcasting information to nearby clients and APs. In practice, when an AP needs to broadcast information, it can send the information to other APs (and through them to their clients) over the backhaul network, while of course directly broadcasting the information to its own clients. On the other hand, when a client needs to broadcast information, it first transmits the information to its AP, which subsequently broadcasts the same to its other clients, as well as send to other APs over the backhaul network. This ensures that all clients and all APs have the correct values of the current state when they make updates. Finally, as shown in Algorithm 1, the amount of exchange information is limited, and each client or AP only needs to broadcast information to APs and clients that are physically close to itself. Hence, our protocol only causes a small overhead on the backhaul connections.

Algorithm 1 Distributed Protocol (DP)

1: **for** $i \in \mathbb{I}$ **do**
2:   $n_i \leftarrow n$, for some arbitrary $n \in \mathbb{N}_i$
3:   broadcast values of n_i and w_i to all $n \in \mathbb{N}_i$
4: **for** $n \in \mathbb{N}$ **do**
5:   $c^n \leftarrow c$, for some arbitrary $c \in \mathbb{C}$
6:   $w^n \leftarrow \sum_{i:n_i=n, n \in \mathbb{N}_i} w_i$
7:   **for** $i : n_i = n, n \in \mathbb{N}_i$ **do**
8:     $\phi_{i,n} \leftarrow w_i/w^n$
9:   broadcast values of c^n and w^n to all $m \in \cup_c \mathbb{M}^{n,c}$ and all $\{i|n \in \mathbb{M}_i\}$
10: **for** $n \in \mathbb{N}$ **do**
11:   $z^n \leftarrow \sum_{m \in \overline{\mathbb{M}}^{n,c^n}} w^m - w^n$
12:   $p^n \leftarrow w^n/(z^n + w^n)$
13:   broadcast the value of z^n to all $m \in \cup_c \mathbb{M}^{n,c}$ and all $\{i|n \in \mathbb{M}_i\}$
14: **for** $v \in \mathbb{I} \cup \mathbb{N}$ **do**
15:   choose arbitrary positive numbers τ_v and θ_v
16: **for** time t such that $\frac{t-\theta_v}{\tau_v}$ is an integer, for some v **do**
17:   $T_t \leftarrow 1/(\log t)^{1-\epsilon}$, for any fixed $\epsilon > 0$
18:   **if** $v \in \mathbb{I}$ **then**
19:     choose n_v randomly, with the probability choosing $n_v = n$ proportional to $e^{\Delta U_v^n(\psi_t)/T(t)}$
20:     broadcast values of n_v and w_v to all $n \in \mathbb{N}_v$
21:     **for** $n \in \mathbb{N}_v$ **do**
22:       $w^n \leftarrow \sum_{i:n_i=n, n \in \mathbb{N}_i} w_i$
23:       **for** $i : n_i = n, n \in \mathbb{N}_i$ **do**
24:         $\phi_{i,n} \leftarrow w_i/w^n$
25:       broadcast values of c^n and w^n to all $m \in \cup_c \mathbb{M}^{n,c}$ and all $\{i|m \in \mathbb{M}_i\}$
26:     **for** $m \in \mathbb{M}_v$ **do**
27:       $z^m \leftarrow \sum_{o \in \overline{\mathbb{M}}^{m,c^m}} w^o - w^m$
28:       $p_m \leftarrow w^m/(z^m + w^m)$
29:       broadcast the value of z^m to all $o \in \cup_c \mathbb{M}^{m,c}$ and all $\{j|n \in \mathbb{M}_j\}$
30:   **else if** $v \in \mathbb{N}$ **then**
31:     choose $c(v)$ randomly, with the probability choosing $c(v) = c$ proportional to $e^{\Delta U_v^c(\psi_t)/T(t)}$
32:     broadcast values of $c(v)$ and w^v to all $m \in \cup_c \mathbb{M}^{v,c}$ and all $\{i|v \in \mathbb{M}_i\}$
33:     **for** $m \in \cup_c \mathbb{M}^{v,c}$ **do**
34:       $z^m \leftarrow \sum_{o \in \mathbb{M}^m(\psi)} w^o - w^m$
35:       $p_m \leftarrow w^m/(z^m + w^m)$
36:       broadcast the value of z^m to all $o \in \cup_c \mathbb{M}^{m,c}$ and all $\{j|n \in \mathbb{M}_j\}$

VII. SIMULATION RESULTS

We have simulated both DP and Greedy algorithms to compare their performances against other state-of-the-art solutions. We present both simulation results for the server-centric scheme and those for the fully distributed scheme.

We first introduce the model of channel characteristics in our simulation. We use the ITU path loss model [34] to compute the received signal strength between two devices. If two devices operate in a band with frequency f_c and are d meters apart, the received signal strength of a device by the other is proportional to $\frac{1}{f_c^2 d^\alpha}$, where α is the path loss coefficient and is set to be 3.5.

We adopt the simulation settings in [5], which models 802.11b channels, as the base case and compute the characteristics of other channels accordingly. 802.11b operates in the 2.4 GHz band with bandwidth 22 MHz. The bit rate between a client and an AP is 11 Mbps if the distance between them is within 50 meters, 5.5 Mpbs within 80 meters, 2 Mpbs within 120 meters, and 1 Mps

---

Algorithm 2 Greedy Policy (Greedy)

1: This approach is identical to Algorithm 1 except that, in Step 19 and Step 31, the states of clients and APs are chosen by $n(v) \leftarrow \arg\max_n \Delta U_v^n(\psi_t)$ and $c^n \leftarrow \arg\max_c \Delta U_v^c(\psi_t)$, respectively.

---

within 150 meters. The *maximum transmission range* of 802.11b channels is hence 150 meters. We assume that two APs interfere with each other if the received signal strength of one AP by the other is above the carrier sense threshold, which, as the settings in ns-2 simulator, is set to be 23.42 times smaller than the received signal strength at a distance of the maximum transmission range. Using the ITU path loss model, two APs interfere with each other if they are within the *interference range,* which is $150 \times (23.42)^{\frac{1}{3.5}} = 369$ meters for 802.11b channels.

For channels other than 802.11b channels, we assume that each of them can support four different data rates, corresponding to the four data rates of 802.11b channels. Values of each supported data rate is proportional to the bandwidth of the channel. The transmission range of each data rate, as well as the interference range, is computed so that the received signal strength at the boundary of the range is the same as that at the boundary of its counterpart in 802.11b channels. For example, consider a channel that operates in frequency 4 GHz with bandwidth 44 MHz. The bit rate between a client and an AP is $11 \times \frac{44}{22} = 22$ Mbps if the distance between them is within $50/(\frac{4}{2.4})^{\frac{2}{3.5}} = 37.34$ meters, 11 Mbps within 59.75 meters, 4 Mpbs within 89.62 meters, and 2 Mbps within 112.03 meters. The interference range of this channel is 275.59 meters.

For the server-centric scheme, we compare our algorithms, DP and Greedy, against policies that use state-of-the-art techniques for solving the Client Association Problem and the Channel Selection Problem. Specifically, we compare with [4], which proposes a distributed algorithm for performing channel selection so as to minimize total interference among APs. For the Client Association Problem and the Scheduling Problem, we compare with two techniques. The first technique uses a Wifi-like approach where clients are associated with the closest AP and the AP schedules clients so that the throughput of each client is the same. The protocol that applies both [4] and the Wifi-like approach is called *MinInt-Wifi.* The other technique is one that is proposed in [5], which, under a fixed solution of the Channel Selection Problem, is a centralized algorithm that aims to find the joint optimal solution to the Client Association Problem and the Scheduling Problem that achieves weighted proportional fairness. This technique first relaxes the Client Association Problem by assuming that each client can be associated with more than one APs and formulates the problem as a convex programming problem. It then rounds up the solution to the convex programming problem and finds a solution to the Client Association Problem where

each client is associated with only one AP. For ease of comparison, we use the solutions to the relaxed convex programming problem, which is indeed an upper-bound on the performance of [5]. The protocol that applies both [4] and [5] is called *MinInt-PF.* The Channel Access Problem is then solved by the optimal solutions based on the resulting solutions of MinInt-Wifi and MinInt-PF, respectively.

In each of the following simulations, we initiate the system by randomly assigning channels to each radio of APs. Each client is initially associated with the closest radio, with ties broken randomly. The system then evolves according to the evaluated policies. We compare the policies on two metrics: the weighted sum of the logarithms of throughput for clients, $\sum_{i \in \mathbb{I}} w_i \log r_i$, and the total weighted throughput $\sum_{i \in \mathbb{I}} w_i r_i$. All reported data are the average over 20 runs. We first discuss the simulation results for a simple system consisting of 3 APs and 2 different channels. While this system is simplistic, it offers insights on the behavior of each policy. We then discuss the simulation results for a larger system where the list of available channels is gathered from a real-world scenario.

Consider a system with 3 APs, each with one radio, that are separated by 75 meters and are located at positions (0,0), (75,0), and (150,0). There are 16 clients, all with weights 1.0, and the ith client is located at position $(35 + 5i, 0)$. We consider two settings for channels: one that with only one 802.11b channel and the other with one 802.11b channel and a channel that operates at frequency 16 GHz with bandwidth 50 MHz. This channel can support higher data rates but has smaller transmission and interference ranges.

Simulation results are shown in Fig. 2. DP and Greedy outperform MinInt-Wifi and MinInt-PF in both evaluated metrics under both 1-channel and 2-channel settings. For the case where there is only one channel, the solutions of the Channel Selection Problem does not have any influence on the results. MinInt-PF does not have good performance because it distribute clients equally to all three APs, which leads to serious contention and collisions within the network. MinInt-Wifi also suffers from the same problem. On the other hand, under DP, all clients are associated with the AP located at (75,0) and therefore contention is avoided. This result suggests that a desirable algorithm for the Client Association Problem also needs to jointly consider the effects on both the Scheduling Problem and the Channel Access Problem.

For the case where there are two channels, both MinInt-Wifi and MinInt-PF select the APs at (0,0) and (150,0) to operate in the channel at frequency 16 GHz with bandwidth 50 MHz and the AP at (75,0) to operate in the 802.11b channel. This selection is the only one that results in no interference within the network. On the other hand, DP selects the APs at (0,0) and (150,0) to operate in the 802.11b channel and the AP at (75,0) to operate in the other channel. While this selection results in interference between the APs at (0,0) and (150,0), DP actually achieves better performance in both metrics.
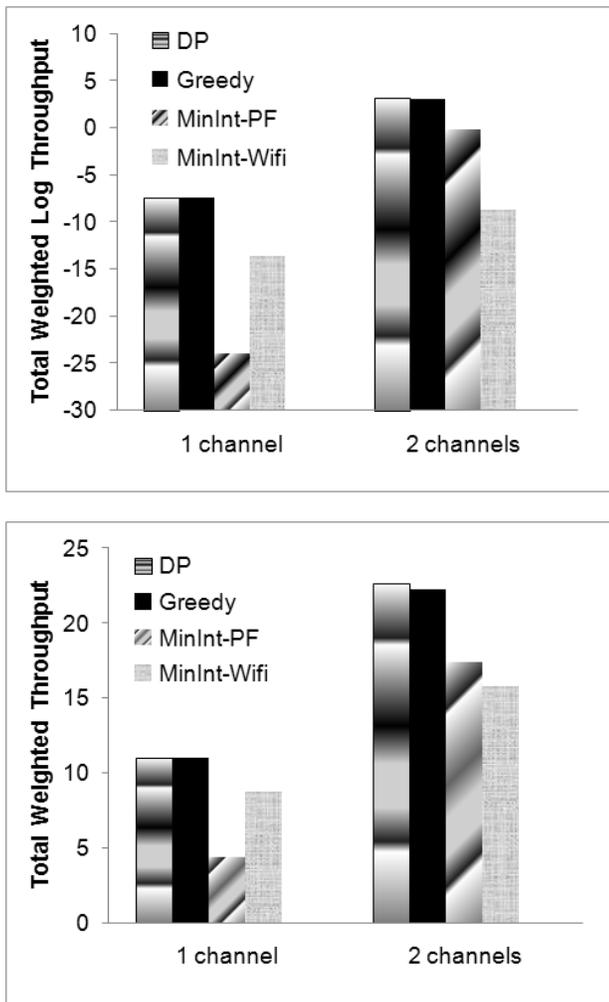
Fig. 2: Performance comparison for a 3-AP system.

id	frequency	bandwidth	id	frequency	bandwith
A	524 MHz	12 MHz	E	659 MHz	6 MHz
B	593 MHz	6 MHz	F	671 MHz	6 MHz
C	608 MHz	12 MHz	G	683 MHz	6 MHz
D	641 MHz	6 MHz			

TABLE I: List of white spaces in New York City.

Fig. 3: Performance comparison for a larger system with unweighted clients in the server-centric scheme.

This is because, in our setting, most clients are gathered around the AP at (75,0) and thus an optimal solution should allow the AP at (75,0) to operate in a channel with higher data rates. This shows that an algorithm that aims to minimize interference among APs may not be optimal because it fails to consider the geographical distribution of clients. Further, although the performance of Greedy is suboptimal, which is because the Client Association Problem and the Channel Selection Problem are non-convex, it is actually close to that of DP and is much better than those of MinInt-Wifi and MinInt-PF.

Next, we consider a larger system, consisting of 16 APs that are placed on a 4 by 4 grid. Adjacent APs are separated by 300 meters. There are 16 clients uniformly distributed in each of the two sectors $[0, 300] \times [0, 300]$ and $[600, 900] \times [600, 900]$; There are 9 clients uniformly distributed in each of the two sectors $[0, 300] \times [600, 900]$ and $[600, 900] \times [0, 300]$. We consider the TV white spaces available in New York City [31]; The list of available channels is shown in Table I. We consider two settings: an unweighted setting where all clients have weights 1.0, and a weighted setting where clients within the region

$[0, 300] \times [0, 900]$ have weights 1.5 and clients outside this region have weights 0.5.

Simulation results for different numbers of radios that each AP has are shown in Fig. 3 and in Fig. 4 for the unweighted setting and the weighted setting, respectively. For both the unweighted and weighted settings, MinInt-Wifi and MinInt-PF are far from optimum. The total weighted throughputs achieved by the two policies are usually less than one-third of those achieved by DP under both settings, especially when the number of radios is large. The performance of Greedy is close to optimum, whose weighted total throughputs are more than 82% of those by DP, under all the settings considered.
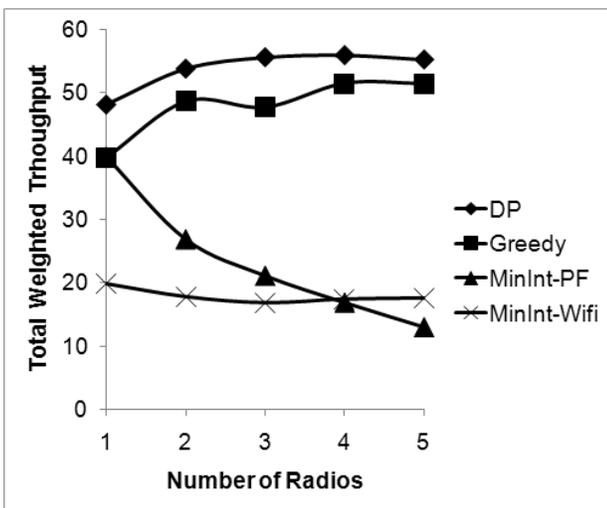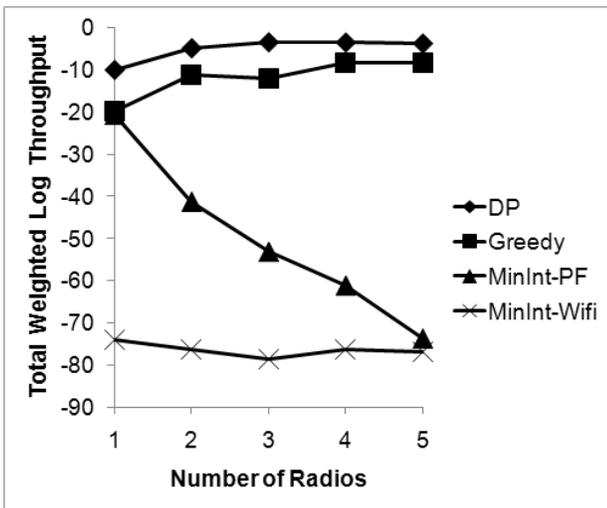
Fig. 4: Performance comparison for a larger system with weighted clients in the server-centric scheme.

Fig. 5: Performance comparison for a larger system with unweighted clients in the fully distributed scheme.

For the fully distributed scheme, we also compare our algorithms, DP and Greedy, against MinInt-Wifi. Since [5] is not directly applicable to the fully distributed scheme, we cannot compare our algorithms against MinInt-Wifi.

We consider the same system as that considered in the simulations for the server-centric scheme. This system consists of 16 APs, 50 clients, and 7 channels. We consider both the unweighted setting and weighted setting.

Simulation results for the fully distributed scheme are shown in Fig. 5 and Fig. 6. In both settings, both DP and Greedy outperform MinInt-Wifi, especially when the number of radios that each AP has is large. Thus, for the fully distributed scheme, a solution that jointly consider all the three components, namely, the Channel Access Problem, the Client Association Problem, and the Channel Selection Problem, is also needed. On the other hand, while Greedy is suboptimal, its performance is close to that achieved by DP. The total weighted throughputs of Greedy are at least 80% of those achieved by DP, under all settings considered here.
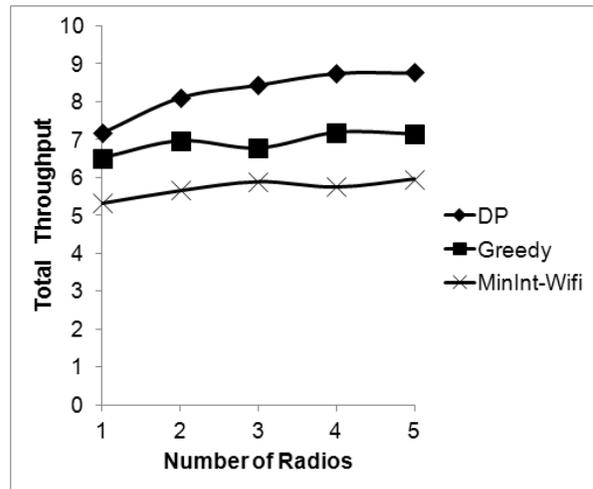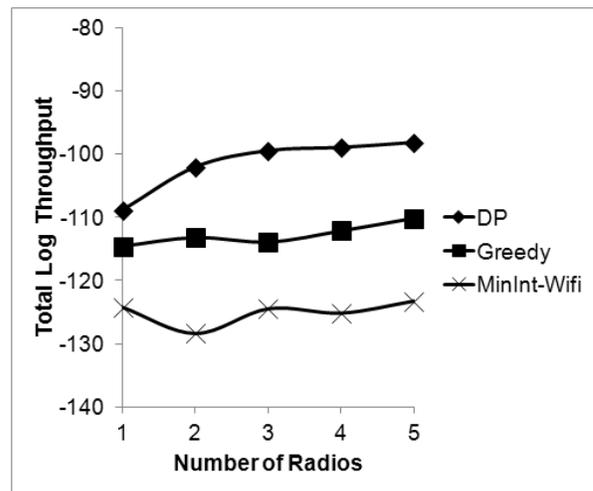
Finally, we evaluate the convergence speed of DP and Greedy. We consider the server-centric scheme and use the same settings as those in Fig. 3 and Fig. 4. Simulation results are shown in Fig. 7, where we present the results for both the cases of when the clients are weighted and are unweighted. In both schemes, Greedy converges very fast. On the other hand, DP converges slowly. Moreover, with a limited number of updates, DP does not always have better performance than Greedy. This suggests that Greedy may be preferable when convergence rate and system stability are important.

VIII. CONCLUSION

We have studied the problem of achieving weighted proportional fairness in multi-band wireless networks. We have considered a system that consists of several APs and clients operating in a number of available channels, accounting for interference among APs and heterogeneous characteristics of different channels. We have identified that the problem of achieving weighted proportional fairness in such a system involves four important components: client scheduling, channel access, client
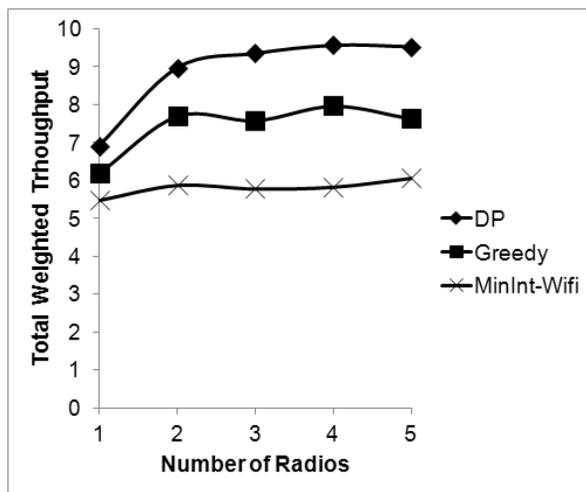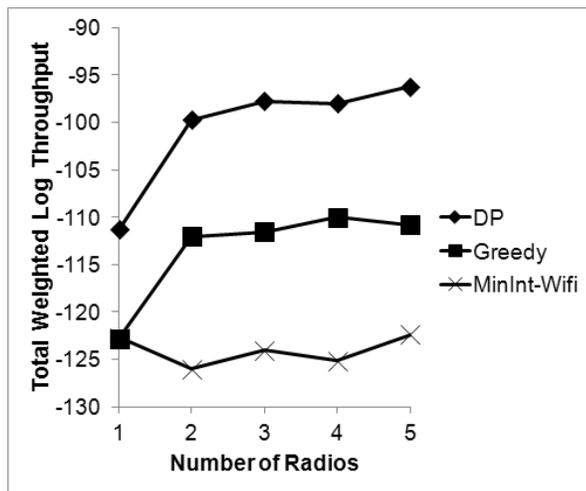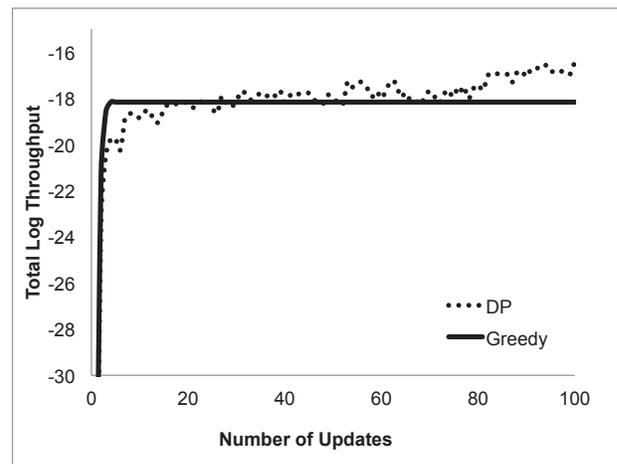
Fig. 6: Performance comparison for a larger system with weighted clients in the fully distributed scheme.

(a) A system with unweighted clients.

(b) A system with weighted clients.

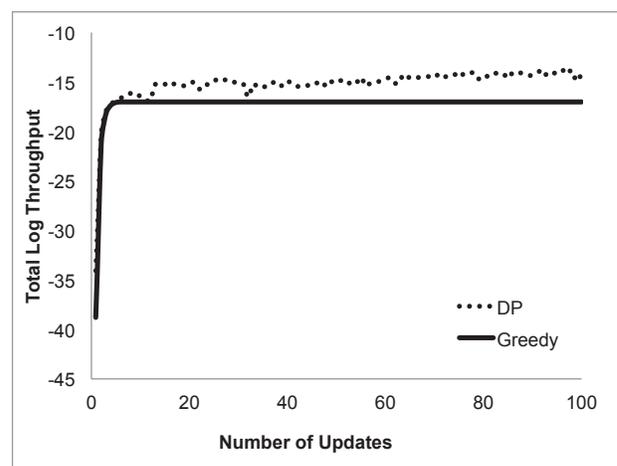Fig. 7: Performance comparison for a larger system with weighted clients in the fully distributed scheme.

association, and channel selection. We have proposed a distributed protocol that jointly considers the four components and achieves weighted proportional fairness. We have also derived a greedy policy based on the distributed protocol that is easier to implement. Simulation results have shown that the distributed protocol outperforms state-of-the-art techniques. The total weighted throughputs achieved by the distributed protocol can be thrice as large as state-of-the-art techniques. Simulation results have also shown that, while being suboptimal, the performance of the greedy policy is actually close to optimum quite often.

REFERENCES

[1] C.-K. Toh, *Ad hoc mobile wireless networks: protocols and systems*. Prentice-Hall, New Jersey, 2001.

[2] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, "Femtocell networks: a survey," *IEEE Commun. Mag.*, vol. 46, no. 9, pp. 59–67, 2008.

[3] H. Claussen, L. T. Ho, and L. G. Samuel, "An overview of the femtocell concept." *Bell Labs Tech. Journal*, vol. 13, no. 1, pp. 221–245, Mar. 2008.

[4] B. Kauffmann, F. Baccelli, A. Chaintreau, V. Mhatre, K. Papagiannaki, and C. Diot, "Measurement-based self organization of interfering 802.11 wireless access networks," in *Proc. of INFOCOM'07*.

[5] L. Li, M. Pal, and Y. Yang, "Proportional fairness in multi-rate wireless LANs," in *Proc. of INFOCOM*, 2008.

[6] J. Deng and Z. Haas, "Dual busy tone multiple access (DBTMA): A new medium access protocol for packet radio networks," in *Proc. of IEEE ICUPC Conf.*, 1998.

[7] R. Rozovsky and P. Kumar, "SEEDEX: A MAC protocol for ad hoc networks," in *Proc. of ACM MobiHoc*, 2001, pp. 67–75.

[8] A. Veres, A. T. Campbell, M. Barry, and L. Sun, "Supporting service differentiation in wireless packet networks using distributed control," *IEEE JSAC.*, vol. 19, no. 10, pp. 2081–2093, Oct. 2001.

[9] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "A spatial reuse Aloha MAC protocol for multihop wireless mobile networks," in *Proc. of 41st Annual Allerton Conf. on Commun., Control, and Computation*, 2003, pp. 1655–1664.

[10] P. Gupta, Y. Sankarasubramaniam, and A. L. Stolyar, "Random-access scheduling with service differentiation in wireless networks," in *Proc. of INFOCOM*, 2005.

[11] L. Jiang and J. Walrand, "A distributed csma algorithm for throughput and utility maximization in wireless networks," *IEEE/ACM Trans. Netw.*, vol. 18, no. 3, pp. 960–972, Jun. 2010.

[12] K. Kar, S. Sarkar, and L. Tassiulas, "Achieving proportionally fair rates using local information in aloha networks," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1858–1862, 2004.

[13] P. Gupta and A. L. Stolyar, "Optimal throughput allocation in general random-access networks," in *Proc. of CISS*, 2006.

[14] P. Gupta and A. Stolyar, "Throughput region of random-access networks of general topology," *Information Theory, IEEE Transactions on*, vol. 58, no. 5, pp. 3016–3022, 2012.

[15] A.-H. Mohsenian-Rad, V. Wong, and R. Schober, "Optimal sinr-based random access," in *INFOCOM, 2010 Proceedings IEEE*, 2010, pp. 1–9.

[16] P. Bahl, R. Chandra, and J. Dunagan, "SSCH: Slotted seeded channel hopping for capacity improvement in IEEE 802.11 ad-hoc wireless networks," in *Proc. of ACM MobiCom*, 2004.

[17] J. So and N. Vaidya, "Multi-channel MAC for ad hoc networks: Handling multi-channel hidden terminals using a single transceiver," in *Proc. of ACM MobiHoc*, 2004.

[18] Y. Bejerano, S.-J. Han, and L. Li, "Fairness and load balancing in wireless LANs using association control," in *Proc. of ACM MobiCom*, 2004, pp. 315–329.

[19] M. Chen, S. C. Liew, Z. Shao, and C. Kai, "Markov approximation for combinatorial network optimization," in *INFOCOM, 2010 Proceedings IEEE*, march 2010, pp. 1 –9.

[20] C. Chen and F. Baccelli, "Self-optimization in mobile cellular networks: Power control and user association," in *Proc. of IEEE ICC*, 2010.

[21] C. Chen, F. Baccelli, and L. Roullet, "Joint optimization of radio resources in small and macro cell networks," in *Proc. of IEEE VTC*, 2011.

[22] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, and A. Viterbi, "CDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 70–77, Jul. 2000.

[23] M. Andrews, K. Kumaran, K. Ramanan, A. L. Stolyar, R. Vijayakumar, and P. Whiting, "Providing quality of service over a shared wireless link," *IEEE Commun. Mag.*, vol. 39, no. 2, pp. 150–154, Feb. 2001.

[24] M. Andrews, L. Qian, and A. L. Stolyar, "Optimal utility based multi-user throughput allocation subject to throughput constraints," in *Proc. of INFOCOM*, 2005.

[25] A. Stolyar and H. Viswanathan, "Self-organizing dynamic fractional frequency reuse in OFDMA systems," in *Proc. of INFOCOM*, 2008.

[26] ——, "Self-organizing dynamic fractional frequency reuse for best-effort traffic through distributed inter-cell coordination," in *Proc. of IEEE INFOCOM*, 2009.

[27] M. Andrews, V. Capdevielle, A. Feki, and P. Gupta, "Autonomous spectrum sharing for mixed LTE femto and macro cells deployments," in *Proc. of INFOCOM WiP*, 2010.

[28] I.-H. Hou and C. S. Chen, "An energy-aware protocol for self-organizing heterogeneous lte systems," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 5, pp. 937–946, 2013.

[29] S. Borst, M. Markakis, and I. Saniee, "Nonconcave utility maximization in locally coupled systems, with applications to wireless and wireline networks," *Networking, IEEE/ACM Transactions on*.

[30] P. Bahl, R. Chandra, T. Moscibroda, R. Murty, and M. Welsh, "White space networking with wi-fi like connectivity," in *Proc. of ACM SIGCOMM*, 2009.

[31] S. Deb, P. Gupta, C. Kanthi, and V. Srinivasan, "An agile and efficient MAC for wireless access over TV white spaces," *Bell Labs Tech. Memo.*, Jul. 2010.

[32] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. on information theory*, vol. 46, no. 2, pp. 388–404, 2000.

[33] P. Brémaud, *Markov Chains, Gibbs Field, Monte Carlo Simulation and Queues*. Springer-Verlag, 1999.

[34] "Propagation data and prediction methods for the planning of indoor radio communication systems and the radio local area networks in the frequency range 900 MHz to 100 GHz," *ITU-R Recommendations*, 2001.
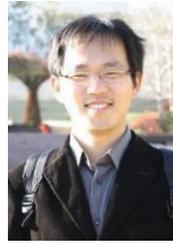
I-Hong Hou (S10-M12) received the B.S. in Electrical Engineering from National Taiwan University in 2004, and his M.S. and Ph.D. in Computer Science from University of Illinois, Urbana-Champaign in 2008 and 2011, respectively.

In 2012, he joined the department of Electrical and Computer Engineering at the Texas A&M University, where he is currently an assistant professor. His research interests include wireless networks, wireless sensor networks, real-time systems, distributed systems, and vehicular ad hoc networks.

Dr. Hou received the C.W. Gear Outstanding Graduate Student Award from the University of Illinois at Urbana-Champaign, and the Silver Prize in the Asian Pacific Mathematics Olympiad.

Piyush Gupta (F11) received the Bachelor of Technology degree in electrical engineering from the Indian Institute of Technology, Bombay, in 1993, the Master of Science degree in computer science and automation from the Indian Institute of Science, Bangalore, in 1996, and the Ph.D. degree in electrical and computer engineering from the University of Illinois, Urbana-Champaign, in 2000.

From 1993 to 1994, he was a design engineer at the Center for Development of Telematics, Bangalore. Since September 2000, he has been a Member of Technical Staff in the Mathematics of Networks and Communications Research Department at Alcatel-Lucent, Bell Laboratories, Murray Hill, NJ. His research interests include wireless networks, network information theory, and learning and adaptive systems.