Truthful and Non-Monetary Mechanism for Direct Data Exchange

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***Abstract*—In recent years, direct data transfer between mobile devices has attracted significant interest from the research community. In this paper, we focus on the scenario in which a group of clients need to exchange a set of files over a lossless broadcast channel. Initially, each client has a subset of files available to it as a side information and needs to obtain the some of the other files in the set. The clients employ the network coding technique to increase efficiency. We assume that each client is *selfish*, i.e., its goal is to minimize its own transmission rate while trying to receive as much information as possible from other clients. The process is mediated by a one of the clients, referred to as a *broker*, that obtains clients' bids and determines their transmissions rates.**

Our goal is to design a *truthful* mechanism that provides incentives for the clients to report the true values of the requested packets. While mechanisms based on the Vickery-Clarke-Groves (VCG) auction are known to be truthful, they involve monetary transactions, and hence are not suitable for the problem at hand. Accordingly, we propose a simple non-monetary algorithm that guarantees truthfulness. Our analytical results and a numerical study indicate that the total utility of all clients under our algorithm is very close to the optimum.

I. Introduction

There is a growing interest in wireless peer to peer (P2P) and device to device (D2D) communication between mobile clients. In a typical scenario, the clients form a small ad-hoc network to exchange the data directly with each other. For example, a small number of mobile devices can exchange the data over a local network (such as Wi-Fi or Bluetooth) to reduce delays and minimize the load on more expensive long-range cellular networks. Communication over a local network has many advantages, such as reduced power consumption and lower delays.

The advantages of wireless P2P/D2D networks have been demonstrated in several recent studies (see e.g., [1–4]). Wireless networks are different from the traditional wired networks due to the broadcast nature of wireless spectrum. In particular, in wireless networks some of the neighboring clients can benefit from a single broadcast transmission hence data exchange typically involves several neighbors rather than only two as in wired P2P networks.

We focus on data exchange between a group of *selfish* wireless clients. Each client initially holds a subset of files and is interested in some of the files held by other clients. The clients use the network coding technique to increase the efficiency of data exchange. For example, Fig. 1 shows three clients c_1, c_2, and c_3 that need files p_1, p_2, and p_3 and have file sets $\{p_2, p_3\}, \{p_1, p_3\}, \{p_1, p_2\}$ available to them as a side information, respectively, i.e., client c_1 holds files p_2 and p_3, client c_2 holds files p_1 and p_3, and client c_3 holds files p_1 and p_2. Suppose that each of the files can be delivered to all the clients with one transmission. Without network coding, at least three transmissions are necessary to satisfy the demands of all clients. By using the network coding technique, all clients can be satisfied by just two transmissions. Indeed, it is easy to verify that if client c_1 transmits a linear combination of p_2 and p_3, and client c_2 transmit p_1, all three clients will be able to decode required packets.

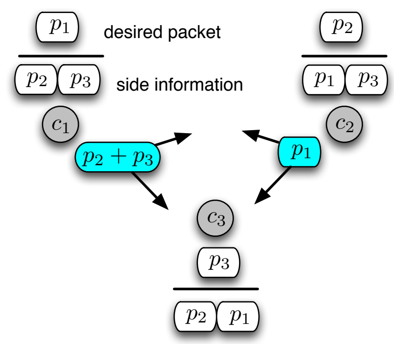

Fig. 1. Coded information exchange between clients c_1, c_2, c_3.

While the problem of minimizing the number of transmissions with network coding has been considered in the previous studies [8–10], the problem of mechanism design for the settings with selfish clients remained open. Indeed, a selfish client might choose to become a "free rider" by choosing not to transmit or make as few transmissions as possible. Indeed, in some cases such clients will be able to decode all packets they need without making a single transmission (e.g., client c_3 in Fig. 1). This, in turn, could affect the data exchange process with negative consequences for all clients. Furthermore, different clients might have different internal values for the packets they request and the clients with low valuations are less likely to participate in the data exchange process. Since the clients are selfish, they may not reveal their internal valuation to other clients if this does not benefit them. Accordingly, in this paper, we focus on design and analysis of a *truthful* mechanism under which each client optimizes its utility by reporting the true valuations of the required packets.

We focus on a setting in which data exchange is managed by a distinguished client (referred to as a *broker*). In the beginning of data transfer the broker obtains a *bid* from each client which specifies its maximum *transmission rate* (i.e., the ratio of transmitted and received packets). After receiving the bids, the broker determines the transmission rate of each client as well as the network coding scheme. In a truthful mechanism, the clients have incentive to bid the maximum transmission rate that corresponds to their valuations of the packets. A popular truthful mechanism is the Vickery-Clarke-Groves (VCG) mechanism [11–13]. However, VCG mechanism and its variations involve monetary transactions between clients and the broker, hence they cannot be applied to the problem at hand.

In this paper, we propose a truthful non-monetary mechanism for the broker. We focus on the case in which each client needs a unique file and each transmission can be received by all clients in the system (i.e., all the clients are in close proximity of each other). We then show that our mechanism can be extended for a setting where a transmission of a client can only be received by some of clients in the group (i.e., some clients are too far away to receive the transmissions from each other) and several clients require the same file. Since our mechanism is non-monetary and does not require additional infrastructure, it can be easily implemented in practical settings. We further demonstrate that this mechanism can be implemented in a fully distributed fashion without the presence of the broker.

We further study the performance of our mechanism in terms of the *social welfare*, i.e., the total value of the decoded files minus the total cost of all transmissions. We establish an upper bound of $\mathcal{O}(\log N)$ on the difference between the optimal social welfare and the social welfare achieved by our algorithm, where N is the number of clients in the group. Through simulations, we also demonstrate that this difference is actually very small for practical settings.

The rest of this paper is organized as follows. The system model is detailed in Section II. In Section III, we examine the feasibility condition, while a VCG-based mechanism is discussed in Section IV. Our mechanisms is discussed in Section V. The optimality of the proposed solution is analyzed in Section VI. Section VII extends the proposed mechanism for a more general setting. Numerical results are presented in Section VIII. Finally, conclusions appear in Section IX.

II. System overview

Consider a wireless network that consists of N clients $\Lambda = \{c_1, \cdots, c_N\}$ that need to exchange a set P of files. We assume that client c_i needs file $p_i \in P$, and has side information $H_i \subseteq P$ available to it. We can assume without loss of generality that each client requires a single file since a client that requires more than one file can be substituted by multiple clients that share the same side information set. Each client c_i has a private value v_i, where $0 \leq v_i \leq 1$, for file p_i that captures its benefit of receiving p_i.

Clients are able to obtain the needed files via data exchanges over a wireless broadcast channel. The transmission process between clients is mediated by one of the clients, referred to as a *broker*. The broker is only conceptual since we will show in Section V that the transmission process can be determined in a fully distributed fashion. We assume that files are of the same size and each contains Z packets.

We consider that clients need to pay some *transmission cost* for the packets they transmit. Specifically, we define the *upload ratio* r_i of client c_i as (number of packets transmitted by $c_i)/Z$. The transmission cost of client c_i is then assumed to be $C(r_i)$, where $C(\cdot)$ is a non-decreasing and convex function with $C(0) = 0$ and $C(1) = 1$. The transmission cost of a client can be interpreted as, for example, the cost of battery power or the cost of channel access for transmitting packets. We denote by $R = \{r_1, \cdots, r_N\}$ the array that contains the upload ratios of all clients.

In the beginning of the data exchange, each client is required to submit a bid b_i to the broker. The bid specifies the maximum transmission cost the client is willing to incur in exchange for file p_i. We denote by $B = \{b_1, \cdots, b_N\}$ the array that contains the bids of the clients. Based on the clients' bids B, the broker decides for each client c_i a *transmission schedule* that consists of the upload ratio r_i, as well as the combination of the packets client c_i needs to transmit. The upload ratio for client c_i must satisfy $C(r_i) \leq b_i$.

Given the transmission schedules, we can determine if client c_i is able to decode the required file p_i. We defined by χ_i the indicator function to specify if client c_i can decode p_i. The *net utility* $u_i(B)$ of c_i can then be written as $u_i(B) = v_i \cdot \chi_i - C(r_i)$. Note that the net utility depends on the bids of other clients as well as the broker's mechanism for determining transmission schedules.

The algorithm for determining the transmission schedules is known to all the parties. Each client c_i is considered to be selfish and chooses its best bidding policy b_i that maximizes its utility $u_i(B)$. We say that a mechanism is *truthful* if each client maximizes its own net utility by choosing $b_i = v_i$.

Definition 1. Let $B_{-i} = B \backslash \{b_i\}$. A mechanism is truthful if, under it, $u_i(v_i, B_{-i}) \geq u_i(b_i, B_{-i})$ for all b_i and B_{-i}.

In this paper, our goal is to design a truthful mechanism that identifies the transmission schedules. The performance of the proposed mechanism is further evaluated in terms of the *total social welfare*, defined as $\sum_{i=1}^{N} v_i \cdot \chi_i - \sum_{i=1}^{N} C(r_i)$, which is the sum of net utilities over all clients.

III. Feasibility of the transmission schedule

The transmission schedule decided by the broker consists of two parts: the upload ratio r_i for each client c_i, and the combinations of the packets that c_i has to transmit. When the vector R of the upload ratios is given, one important question is whether there exists a transmission schedule under which a subset $S \subseteq \Lambda$ of clients can successfully decode their needed files. We establish a sufficient condition for the existence of such a schedule.

Lemma 1. *Assume that $p_i \neq p_j$, for all $i \neq j$. Given a vector R of upload ratios and a subset $S \subseteq \Lambda$ with $r_i = 0$ for all $c_i \notin S$. Suppose that there exists a partition of $S = S_1 \cup S_2 \cup \cdots \cup S_k$ with $S_m \cap S_n = \phi$, for all $m \neq n$, such that, for each $S_m \in S$ and for each $c_i \in S_m$, we have*

1) *$\{p_i\} \in H_j$, for all $c_j \in S_m$, and $j \neq i$, and*
2) *$\sum_{c_j \in S_m \backslash \{c_i\}} r_j \geq 1$.*

Then there exists a transmission schedule such that

i) *The upload ratio of each client c_i is r_i.*
ii) *All clients in S are able to decode their respective needed files.*
iii) *None of the clients outside S can decode their needed files.*

Proof: We prove this lemma by constructing a transmission schedule that satisfies the conditions of the lemma. Let client $c_i \in S_m$ transmit Zr_i coded packets, each containing a linear combination of one packet from each of the files $\{p_j : c_j \in S_m \backslash \{c_i\}\}$. This can be achieved because $\{p_j : c_j \in S_m \backslash \{c_i\}\} \subseteq H_i$ by property (1).

Now, for a client $c_j \in S_m$, it receives Zr_i coded packets from c_i, for each other $c_i \in S_m$. Each of these coded packets contains a linear combination of one packet from each of $\{p_k : c_k \in S_m \backslash \{c_i\}\}$. Since c_j have the files $\{p_k : c_k \in S_m \backslash \{c_i, c_j\}\}$ as its side information, by property (1), it can employ Gaussian elimination to obtain Zr_i packets of the file p_j, for each $c_i \in S_m$. Thus, c_j obtains a total number of $\sum_{i \in S_m, i \neq j} Zr_i \geq Z$ packets of p_j, and successfully receives the whole file p_j.

On the other hand, in the above transmission schedule, no clients transmit any packets that involve $\{p_j : c_j \notin S\}$. Therefore, none of the clients outside S can receive their needed files. ∎

Given a vector R of upload ratios and a subset $S \subseteq \Lambda$ with $r_i = 0$, for all $c_i \notin S$, if the condition in Lemma 1 is satisfied, we say (R, S) is *feasible*. The notion of Lemma 1 is that if (R, S) is feasible, then each client $c_i \in S$ scheduled to transmit (i.e. $r_i > 0$) can receive enough amount of packets to recover its needed file p_i (i.e. $\chi_i = 1$).

For the special case where $H_i = P \backslash \{p_i\}$, for all i, Lemma 1 can be greatly simplified and strengthened as follows:

Corollary 1. *Suppose $H_i = P \backslash \{p_i\}$, for all i. Consider a vector R of upload ratios, and a subset $S \subseteq \Lambda$ with $r_i = 0$, for all $c_i \notin S$. Then (R, S) is feasible if and only if*

$$\sum_{c_j \in S \backslash \{c_i\}} r_j \geq 1,$$

for all $c_i \in S$.

Proof: By Lemma 1, it is straightforward to show that the condition is sufficient. To show that it is necessary, assume that there exists $c_i \in S$ with $\sum_{c_j \in S \backslash \{c_i\}} r_j < 1$, then the total number of packets that c_i receives is no larger than $(\sum_{c_j \in S \backslash \{c_i\}} Zr_j) < Z$. Hence, it is impossible for c_i to obtain all packets in p_i. ∎

IV. Infeasibility of VCG-based mechanism

A classical solution for the truthful mechanism design is to apply the Vickery-Clarke-Groves (VCG) mechanism. However, VCG-based mechanisms are not suitable for solving our problem. For clarity, we briefly discuss the design of the VCG mechanism below.

Consider a system where a centralized server decides which set of clients to serve. When it serves a set S of clients, it needs to pay some cost, defined as $cost(S)$. Each client c_i has a secret valuation $\hat{v}_i$ for being served. The *social welfare* of serving S is defined as $\sum_{c_i \in S} \hat{v}_i - cost(S)$. The VCG mechanism then works as follows: First, each client submits its valuations $\hat{b}_i$ to the server. Note that we do not require $\hat{b}_i = \hat{v}_i$, as clients may lie about their valuations. Then, the server does the following steps:

1) It chooses the set S that maximizes $\sum_{i \in S} \hat{b}_i - cost(S)$.
2) It charges each scheduled client c_i an amount of money that equals to $[(\max_{S': c_i \notin S'} \sum_{c_j \in S'} \hat{b}_j - (cost(S'))) - (\sum_{c_j \in S, j \neq i} \hat{b}_j - (cost(S)))]^+$, where $x^+ := \max\{x, 0\}$. The expression for the charge is called the *critical price*, since it is the minimum value that client c_i needs to bid in order to be scheduled in the previous step.

It is well-known that the VCG mechanism is truthful [14]. However, the VCG mechanism involves monetary exchanges, which requires additional infrastructure. Hence, it is not suitable for direct data exchange in wireless ad hoc networks.

One may wonder whether there exists simple non-monetary adaptations of the VCG mechanism to our problem. We can consider one with the following setting: We treat the server as a broker, $cost(S)$ to be the minimum total upload ratios required to make (R, S) feasible. To be more specific, $cost(S) \equiv \min_{R:(R,S) \text{ is feasible}} \sum_i r_i$. With this interpretation, the definitions of social welfare in the VCG mechanism and in our setting are equivalent. In the second step of the VCG mechanism, the sever charges each client some money. One naive adaptation is to treat the charges of clients as the upload ratios in our system, i.e., $r_i \equiv [(\max_{S': c_i \notin S'} \sum_{c_j \in S'} \hat{b}_j - (cost(S'))) - (\sum_{c_j \in S, j \neq i} \hat{b}_j - (cost(S)))]^+$, for all $c_i \in S$. However, with this adaptation, we fix the vector R of upload ratios, and there is no guarantee that (R, S) is still feasible. A numerical example that demonstrates this intuition is shown below.

Example 1. Consider three clients with the identical valuation $v_i = 0.6$, and the side information $H_i = P \backslash \{p_i\}$. Let the cost function be $C(x) = x$. In the first step, the broker decides that the optimal decision is to schedule all three clients. Indeed, by Corollary 1, there exists a transmission schedule with $r_i = 0.5$, for all i, under which all three clients get their needed files, and the social welfare is $(0.6 - 0.5) \times 3 = 0.3$. However, the second step of the VCG-based algorithm results in $r_i = 0.3$ for all i, and, by

Corollary 1, the set of all clients with such upload ratios are no longer feasible. ◀

V. Truthful mechanism

We first focus on the setting that each client c_i has all files other than p_i, i.e. $H_i = P \backslash \{p_i\}$, while the more general model is discussed in Section VII. We propose a mechanism for the broker as follows: First, we sort bids in descending order such that $b_1 \geq b_2 \geq \ldots$. We then find the maximum number $i^* \in \{1, \cdots, N\}$ with $b_{i^*} \geq C(\frac{1}{i^*-1})$. A client whose bid is greater than or equal to b_{i^*} is included in the set S^*, and is scheduled to transmit with the upload ratio $r_i = \frac{1}{|S^*|-1}$. All other clients neither transmit nor receive their needed files. We notice that no monetary exchange occurs during the process. This mechanism is formally stated in Alg. 1.

Algorithm 1: Truthful mechanism

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input : Bids vector B, and side information H_i for all clients

output: Transmission schedules

1 Sorting bids: $b_1 \geq b_2 \geq \cdots \geq b_N$;

2 Find $i^* = \max\{i : b_i \geq C(\frac{1}{i-1})\}$;

3 $S^* \leftarrow \{c_1, \cdots, c_{i^*}\}$;

4 $r_i \leftarrow \frac{1}{|S^*|-1}$ for $c_i \in S^*$;

5 $r_i \leftarrow 0$ for $c_i \notin S^*$;

6 Client $c_i \in S^*$ transmits r_i proportion of the coded file $\sum_{p \in \{p_1, \cdots, p_{i^*}\} \backslash \{p_i\}} p$;

The following lemma is a direct result from Corollary 1.

Lemma 2. *(R, S^*) produced from Alg. 1 is feasible.*

Next, we will show that Alg. 1 is also truthful.

Theorem 1. *Alg. 1 is a truthful mechanism.*

Proof: Consider a client $c_{\hat{i}}$, where the index of $\hat{i}$ is set to be the position of $c_{\hat{i}}$ in Line 1 of Alg. 1 when it bids $b_{\hat{i}} = v_{\hat{i}}$. That is, we have $b_1 \geq b_2 \geq \ldots b_{\hat{i}-1} \geq v_{\hat{i}} \geq b_{\hat{i}+1} \ldots$. Let $S^*(b_{\hat{i}})$ be the set S^* under Alg. 1 when $c_{\hat{i}}$ bids $b_{\hat{i}}$. We consider the following two cases:

Case (1) $c_{\hat{i}} \notin S^*(v_{\hat{i}})$, i.e., $c_{\hat{i}}$ is not scheduled by bidding its true value: By the design of Alg. 1, we have that $v_{\hat{i}} < C(\frac{1}{\hat{i}-1})$, and $b_i < C(\frac{1}{i-1})$ for all $i > \hat{i}$. Also, the net utility of $c_{\hat{i}}$ is 0 when it bids its true value.

Suppose $c_{\hat{i}}$ bids $b_{\hat{i}} \neq v_{\hat{i}}$, we consider three possibilities based on the position of $b_{\hat{i}}$ in Line 1 of Alg. 1: $b_{\hat{i}-1} \geq b_{\hat{i}} \geq b_{\hat{i}+1}$, i.e., the position is the same as that under the true value; $b_k \geq b_{\hat{i}} \geq b_{k+1}$, for some $k < \hat{i}-1$, i.e., the position is higher than that under the true value; and $b_k \geq b_{\hat{i}} \geq b_{k+1}$, for some $k > \hat{i}$.

In the first and second cases, since the positions of all clients c_i with $i > \hat{i}$ are the same as those when $c_{\hat{i}}$ bids its true value, they are still not scheduled by Alg. 1. Hence, we have $|S^*(b_{\hat{i}})| \leq \hat{i}$, and $r_i \geq \frac{1}{\hat{i}-1}$, for all $i \in S^*(b_{\hat{i}})$. The net utility of $\hat{i}$ is then no larger than $\max\{0, v_{\hat{i}} - C(\frac{1}{\hat{i}-1}\}) \leq 0$, as $v_{\hat{i}} < C(\frac{1}{\hat{i}-1})$.

In the third case, client $c_{\hat{i}}$ is now placed on the k^{th} position with $k > \hat{i}$. We have $b_{\hat{i}} \leq b_k < C(\frac{1}{k-1})$. Hence, $c_{\hat{i}}$ is still not scheduled and have net utility 0. In sum, when $c_{\hat{i}} \notin S^*(v_{\hat{i}})$, $c_{\hat{i}}$ cannot improve its own net utility by lying about its $v_{\hat{i}}$.

Case (2) $c_{\hat{i}} \in S^*(v_{\hat{i}})$, i.e., $c_{\hat{i}}$ is scheduled by bidding its true value: Let $i^* = |S^*(v_{\hat{i}})|$. We have $v_{\hat{i}} \geq b_{i^*} \geq C(\frac{1}{i^*-1})$, and the net utility of $c_{\hat{i}}$ is greater or equal to 0. Now, by the design of Alg. 1, we have $b_i < C(\frac{1}{i-1})$ for all $i > i^*$, and each client c_i with $i > i^*$ will not be scheduled regardless of $b_{\hat{i}}$. Therefore, $|S^*(b_{\hat{i}})| \leq i^*$, and $r_i \geq C(\frac{1}{i^*-1})$, for $i \in S^*(b_{\hat{i}})$, regardless of $b_{\hat{i}}$. The net utility of $c_{\hat{i}}$ is then upper-bounded by $v_{\hat{i}} - C(\frac{1}{i^*-1}) \geq 0$, which is its net utility when bidding $b_{\hat{i}} = v_{\hat{i}}$.

Fully consider cases 1 and 2, we conclude that Alg. 1 is a truthful mechanism. ■

In addition to being truthful, we note that Alg. 1 can also be easily implemented in a fully distributed fashion. Instead of relying on the broker, each client simply broadcasts its bid b_i. After receiving the values of all bids, each client determines how many, and what, packets it should transmit by running Alg. 1. Thus, our mechanism can still be implemented without the presence of the broker.

VI. Performance analysis

In this section, the social welfare, defined by $\sum_{i=1}^{N} v_i \cdot \chi_i - \sum_{i=1}^{N} C(r_i)$, is concerned. We will discuss the loss of optimality in terms of social welfare when Alg. 1 is applied, that is, the difference between the social welfare under Alg. 1 and the largest achievable social welfare. In Lemma 3, we study the structure of the maximum social welfare.

Lemma 3. *The optimal solution $\{\chi_1, \cdots, \chi_N\}$ to maximize social welfare $\sum_{i=1}^{N} v_i \cdot \chi_i - \sum_{i=1}^{N} C(r_i)$ is either $\chi_i = 0$ or $\chi_i = 1$ for all i.*

Proof: If, for any subset $S \subseteq \Lambda$, every feasible (R, S) gives rise to a negative social welfare, i.e. $\sum_{c_i \in S} v_i - \sum_{c_i \in S} C(r_i) < 0$, then the optimal solution is $r_i = 0$ for all i.

Suppose that there is a feasible $(R, \hat{S})$, where $\hat{S} \subseteq \Lambda$, such that $\sum_{c_i \in \hat{S}} v_i - \sum_{c_i \in \hat{S}} C(r_i) \geq 0$. By Corollary 1, we have, for each $c_i \in \Lambda$,

$$\sum_{c_j \in \Lambda \backslash \{c_i\}} r_j = \sum_{c_j \in S \backslash \{c_i\}} r_j \geq 1.$$

Hence, (R, Λ) is also feasible, and the social welfare under (R, Λ) is $\sum_{i=1}^{N} v_i - \sum_{c_i \in \hat{S}} C(r_i) \geq \sum_{c_i \in \hat{S}} v_i - \sum_{c_i \in \hat{S}} C(r_i)$. That is, we can construct another transmission schedule with $\chi_i = 1$, for all i, whose social welfare is at least as large as that under $(R, \hat{S})$. This concludes the proof. ■

Corollary 2. *The maximum social welfare is* $\max\{0, \sum_{i=1}^{N} v_i - N \cdot C(\frac{1}{N-1})\}$.

Proof: According to Lemma 3, either $\chi_i = 0$ or $\chi_i = 1$ for all i maximizes the social welfare. If the optimal solution is $\chi_i = 1$ for all i, then the social welfare will be

$$\begin{aligned} & \sum_{i=1}^{N} v_i - \sum_{i=1}^{N} C(r_i) \\ \leq & \sum_{i=1}^{N} v_i - N \cdot C\left(\frac{1}{N} \cdot \sum_{i=1}^{N} r_i\right) \quad (1) \\ \leq & \sum_{i=1}^{N} v_i - N \cdot C\left(\frac{1}{N-1}\right). \quad (2) \end{aligned}$$

We note that (1) is based on the convex cost function $C(.)$, as well as (2) (i.e. $\sum_{i=1}^{N} r_i \geq \frac{N}{N-1}$) comes from the feasibility condition $\sum_{c_j \in \Lambda \setminus \{c_i\}} r_j \geq 1$ for all $c_i \in \Lambda$. The equalities (1) and (2) are achievable when $r_i = \frac{1}{N-1}$ for all $c_i \in \Lambda$; therefore, the maximum social welfare when $\chi_i = 1$ for all i is $\sum_{i=1}^{N} v_i - N \cdot C(\frac{1}{N-1})$. ■

We are ready to study the loss of optimality in terms of social welfare when Alg. 1 is applied.

Theorem 2. *The loss of optimality in terms of social welfare under Alg.1 is bounded by* $\sum_{i=1}^{N-1} C(\frac{1}{i}) + C(1) = \mathcal{O}(logN)$, *when* $b_i = v_i$, *for all* i.

Proof: If the maximum social welfare is 0, then there is no loss of optimality, as the social welfare under Alg. 1 is always non-negative.

Next consider the case when the maximum social welfare is $\sum_{i=1}^{N} v_i - NC(\frac{1}{N-1})$. Sort all clients so that $v_1 \geq v_2 \geq \dots$. Assume that Alg. 1 schedules a set $S^* = \{1, 2, \dots, i^*\}$ of clients. We then have $r_i = \frac{1}{i^*-1}$, for all $i \leq i^*$, and $v_i < C(\frac{1}{i-1})$, for all $i > i^*$. The social welfare under Alg. 1 is $\sum_{i=1}^{i^*} v_i - i^* \cdot C(\frac{1}{i^*-1})$, and the loss of optimality is

$$\begin{aligned} & \sum_{i=1}^{N} v_i - NC(\frac{1}{N-1}) - (\sum_{i=1}^{i^*} v_i - i^* \cdot C(\frac{1}{i^*-1})) \\ \leq & \sum_{i=1}^{N} v_i - \sum_{i=1}^{i^*} v_i + i^* \cdot C(\frac{1}{i^*-1}) \\ = & \sum_{i=i^*+1}^{N} v_i + i^* \cdot C(\frac{1}{i^*-1}) \\ \leq & \sum_{i=i^*+1}^{N} C(\frac{1}{i-1}) + \sum_{i=2}^{i^*} C(\frac{1}{i-1}) + C(1) \\ = & \sum_{i=1}^{N-1} C(\frac{1}{i}) + C(1). \end{aligned}$$

■

VII. Extensions

Thus far, we focused on the setting that every client c_i misses a unique file and has all other files in P. We also assume that each client can receive the transmission from each other. In this section, we discuss some extensions that can be applied to more general scenarios.

A. Some clients cannot exchange files with each other

So far, we have assumed that each client can transmit a file that is useful for another client. This may not be true in some realistic settings. For example, some clients may be too far away from each other, and hence can not receive the transmissions by each other. Also, it is possible that client c_i does not possess the file that another client c_j needs.

To model this scenario, we say that $p_j \in H_i$ if and only if c_i has the file p_j, and c_j can receive transmissions from c_i. It is easy to check that Lemma 1 still holds with this slight modification. We then define a *dependency graph* as follows:

Definition 2. The dependency graph is an undirected graph G defined as follows:

- For each client $c_i \in \Lambda$, there is a corresponding vertex in G.
- For any two clients c_i and c_j such that $p_i \in H_j$ and $p_j \in H_i$, there is an edge between the corresponding vertices.

Based on the dependency graph, we propose a truthful mechanism in Alg. 2.

Algorithm 2: Truthful mechanism for the dependency graph

input : Bids vector B, and side information H_i for all clients
output: Transmission schedules
1 Create the dependency graph G;
2 $S^* \leftarrow \phi$;
3 $r_i \leftarrow 0, \forall i$;
4 **for** $k \leftarrow N$ **to** 1 **do**
5   **while** *there exists a k-clique in G such that $b_i \geq C(\frac{1}{k-1})$ for all corresponding client c_i in the clique* **do**
6         $S^* \leftarrow S^* \cup \{$ all corresponding c_i in the clique$\}$;
7         $r_i \leftarrow \frac{1}{k-1}$ for all corresponding c_i in the clique ;
8   Remove the clique from G ;
9   **end**
10 **end**

In Line 5 of the above algorithm, if there are multiple cliques that satisfy the condition, ties are broken by a predetermined order that is independent of the bids of clients.

Theorem 3. *Alg. 2 produces a feasible* (R, S^*) *and is truthful.*

Proof: Using Lemma 1, we can establish that Alg. 2 produces a feasible (R, S^*) by treating the partitions $S_1, S_2, \dots$ in Lemma 1 as the cliques found in Line 5.

Consider a client $c_{\hat{i}}$, and the (R, S^*) produced by Alg. 2 when it bids $b_{\hat{i}} = \infty$. If $c_{\hat{i}}$ is not scheduled by bidding ∞, it will not be schedule regardless of its bid. Hence, its net utility is always 0. On the other hand, suppose that $c_{\hat{i}}$ is scheduled by bidding ∞, and let k be the size of the clique when $c_{\hat{i}}$ is included in S^* in Lines 5 – 9 of Alg. 2. Since ties are broken by a predetermined order when multiple cliques satisfy the condition in Line 5, $c_{\hat{i}}$ will be included in S^* with a clique of size k as long as $b_{\hat{i}} \geq C(\frac{1}{k-1})$, and, if $b_{\hat{i}} < C(\frac{1}{k-1})$, it will not be included in S^*. Therefore, if $c_{\hat{i}}$ is scheduled, its upload ratio is $\frac{1}{k-1}$, regardless of its actual bid. The net utility of $c_{\hat{i}}$ is then upper-bounded by $\max\{0, v_{\hat{i}} - C(\frac{1}{k-1})\}$, which can be attained by bidding $b_{\hat{i}} = v_{\hat{i}}$. In sum, client $c_{\hat{i}}$ cannot improve its net utility by lying about its true value. ■

B. *Multiple clients miss the same file*

We consider the case that multiple clients require the same file. Assume that we have a set of clients $\{c_{1,1}, \cdots, c_{1,\kappa_1}, c_{2,1}, \cdots, c_{2,\kappa_2}, \cdots, c_{N,1}, \cdots, c_{N,\kappa_N}\}$, where clients $\{c_{i,1} \cdots, c_{i,\kappa_i}\}$ miss the same file p_i with the identical side information $P \backslash \{p_i\}$. We propose Alg. 3, which is a variation of Alg. 1, to resolve the truthfulness of clients. Intuitively, Alg. 3 combines the clients $\{c_{i,1}, \cdots, c_{i,\kappa_i}\}$ to be a super-client c_i (see Line 1) and then uses the same argument of Alg 1. Moreover, the clients $\{c_{i,1}, \cdots, c_{i,\kappa_i}\}$ evenly split the upload ratio $\frac{1}{i^*-1}$ as in Line 5.

Algorithm 3: Truthful mechanism when multiple clients need the same file

input : Bids vector B, and side information H_i for all clients

output: Transmission schedules

1 $b_i \leftarrow \kappa_i \times \min b_{i,j}$ for all i ;

2 Sorting bids: $b_1 \geq b_2 \geq \cdots \geq b_N$;

3 Find $i^* = \max\{i : b_i \geq C(\frac{1}{i-1})\}$;

4 S^* includes $c_{i,j}$ for $i = 1, \cdots, i^*$ and $j = 1, \cdots, \kappa_i$;

5 $r_{i,j} \leftarrow \frac{1}{\kappa_i} \cdot \frac{1}{i^*-1}$ for $c_{i,j} \in S^*$;

6 $r_{i,j} \leftarrow 0$ for $c_{i,j} \notin S^*$;

7 Client $c_{i,j} \in S^*$ transmits $r_{i,j}$ proportion of the coded file $\sum_{p \in \{p_1, \cdots, p_{i^*}\} \backslash \{p_i\}} p$;

It is easy to establish the following theorem, whose proof is omitted as it is very similar to the proof of Thm. 1.

Theorem 4. *Alg. 3 produces feasible (R, S^*) and is truthful.*

VIII. Numerical results

In this section, we numerically study the performance of Alg. 1 of Section V. We evaluate the loss of optimality in terms of social welfare, which can be written as

$$\max\left\{0, \sum_{i=1}^{N} v_i - N \cdot C(\frac{1}{N-1})\right\} - \left(\sum_{i=1}^{i^*} v_i - i^* \cdot C(\frac{1}{i^*-1}\right)$$

Fig. 2 and 3 show the results for $C(x) = x$ and $C(x) = x^2$, respectively, where the triangle symbol represents the average result over 100000 runs, and the circle one is the 10000^{th} largest value of loss of optimality in the 100000 runs.

The experiment setting is as follows. We consider N clients (x-axle), where client c_i requires file p_i and has the side information $H_i = P \backslash \{p_i\}$. The value v_i of file p_i is

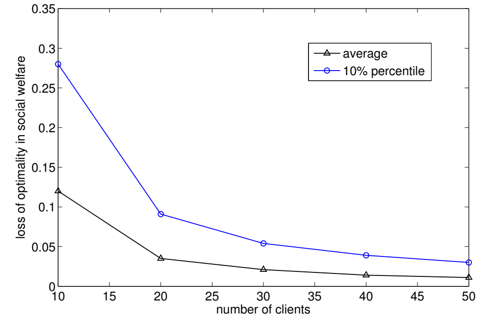

Fig. 2. Loss of optimality in terms of social welfare when the cost function is $C(x) = x$.

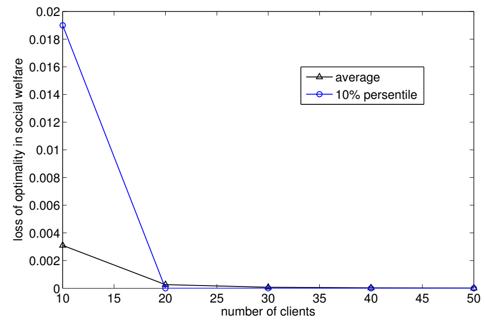

Fig. 3. Loss of optimality in terms of social welfare when the cost function is $C(x) = x^2$.

uniformly distributed between 0 and 1. Let bid $b_i = v_i$ due to the truthfulness. We can observe that the proposed mechanism is close to optimal one as the number of clients gets larger. Note that if $v_N \geq C(\frac{1}{N-1})$, there will be no loss of optimality. Moreover, if we consider $C(x) = x$, then the probability $\mathbb{P}(v_N \geq \frac{1}{N-1}) = (1 - \frac{1}{N-1})^N$ approaches to 1 as $N \rightarrow \infty$. We can then conclude that Alg. 1 performs well in average case when the number of clients is large enough.

IX. Conclusion

In this paper, we considered the wireless data exchange problem from a game theoretical perspective. We showed that the Vickrey-Clarke-Groves (VCG) mechanism is not suitable to resolve our problem. We then proposed a novel non-monetary mechanism that determines the transmission schedules of clients and prove that this mechanism satisfies the truthfulness property. We have evaluated the social welfare of proposed mechanism provided an upper bound on the loss of optimality. Our simulation results further validate that the social welfare of the proposed mechanism is very close to the optimal value.

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