ECEN 689
Special Topics in Data Science for Communications Networks

Nick Duffield
Department of Electrical & Computer Engineering
Texas A&M University

Lecture 18
Measuring link loss rates

- Link transmission rates $\alpha_i$
  - Fraction $1 - \alpha_i$ of packets traversing link $i$ are lost
- Path comprising links $n$ links $\{1, 2, \ldots, n\}$
- Path transmission rate $\beta = \prod_{i=1}^{n} \alpha_i$
  - Fraction $1 - \beta$ of packets are lost somewhere on path
- What is pattern of loss?
  - Uniformly distributed loss: $\alpha_i = \beta^{1/n}$
  - Localized loss: one of the $\alpha_i = \beta$, all other $\alpha_i = 1$
- Can’t distinguish these (or other) loss patterns given only $\beta$
  - Think: identifiability
Network Performance Tomography

• Goal:
  – determine performance of links and routers inside the internet using measurements made from its edge

• Analogy:
  – medical imaging tomography:
    • determine structures inside a body using images formed by radiation that has passed through the body
Medical Imaging Tomography

• Basis:
  – different tissue types absorb radiation differently
    • e.g. bone more strongly absorbent, soft tissue less so

• Aim:
  – build 3-d picture of body by probing with radiation
    • of how strongly it absorbs radiation at different points

• Method:
  – Create multiple 2-d views
    • probe with radiation
    • measure transmitted intensity
      – a point in 2-d view indicates cumulative absorption along line
  – Combine and correlate the views
    • statistical analysis to obtain 3-d picture
      – determine absorption properties at each point
Key Ideas

• Multiple views of an object

• Each view represents cumulative effect on probes due to some property of object

• Correlate views to determine the property at each point in object
Principles of Performance Tomography

• Setting:
  – end-to-end paths traverse many links of the Internet
  – path performance (e.g. packet delay) cumulative over links
• Aim:
  – determine link performance from end-to-end measures
• Application:
  – multiple views of the network
    • measure end-to-end performance with probe packets along different paths
  – correlate measurements on intersecting paths
    • to infer link performance
Network-Independent Measurement

• No participation by network assumed
  – other than the usual forwarding of packets
    • measurement probes are just regular packet

• No administrative access to network needed
  – methods does not use internal operational statistics
    • e.g. no router statistics / packet monitors

• Deployment
  – Fixed measurement infrastructure
    • measurements between dedicated measurement hosts
  – Embedded measurement infrastructure
    • piggybacking on regular traffic, protocols and hosts
Usage Scenarios

• User coalitions
  – users have little access to operational statistics
  – instead: share and correlate measurements
    • across multiple service providers
  – pinpoint performance bottlenecks

• Provider networks
  – diagnose performance degradation
  – in contractor network
    • of dumb network elements that keep no operational statistics
Limits of unicast probing

- Transmission rate $\alpha_i$ on link $i$
  - Link $i$ = directed link $(j,i)$ with terminal node $i$, some $j$
- $A_i =$ path transmission rate from 0 to $i$
  - $A_1 = \alpha_1$; $A_2 = \alpha_1 \alpha_2$; $A_3 = \alpha_1 \alpha_3$
- Assume:
  - Can measure transmission rate to leaf nodes 2 and 3
  - Can not measure transmission rate to interior node 1
- Can we determine $\{\alpha_1, \alpha_2, \alpha_3\}$ from $A_2$ and $A_3$?
- No: 2 knowns, 3 unknowns
- Linear equations
  - $(\log A_2, \log A_3)^T = M. (\log \alpha_1, \log \alpha_2, \log \alpha_3)^T$: $M$ not of full rank
Multicast and Correlation

- Multicast probes good for tomography
  - inherent correlation
    - contribution to end-to-end performance from common path is identical

- How to exploit this correlation for network tomography
Unicast vs. Multicast

- **Setting:**
  - Sending same content to multiple receivers
- **Unicast:**
  - send one packet to each receiver
- **Multicast:**
  - send one packet, replicate as necessary in network
Tree and Loss Model

- **Tree model**
  - Tree $G=(V,L) = (\text{nodes, links})$
  - source multicast probes from root node 0
  - set $R \subset V$ of receiver nodes at leaves

- **Loss model**
  - probe traverses link $k$ with prob. $\alpha_k \in (0,1)$
    - otherwise lost
  - loss independent between links, probes
Probe process

- Stochastic Process $(X_k)_{k \in V}$
- $X_k = 1$ if probe reaches node $k$, 0 otherwise
- $X_0 = 1$
  - probe is present at root at root node
- $\{X_j : j \in d(k)\}$ conditionally independent given $X_k$
  - $d(k) = \text{children of } k$
- $P[X_k = 1 | X_{f(k)} = 1] = \alpha_k$
  - $f(k) = \text{parent of } k$
- $X_k = 0$ whenever $X_{f(k)} = 0$
  - a lost probe remains lost
Performance Tomography in a Simple Tree

- Each probe has one of 4 possible outcomes at leaves
  - \((X_2, X_3) \in \{ (1,1), (1,0), (0,1), (0,0) \} \)
- Theoretical outcome frequencies
  - \(p_{11} = \alpha_1 \alpha_2 \alpha_3\)
  - \(p_{10} = \alpha_1 \alpha_2 (1 - \alpha_3)\)
  - \(p_{01} = \alpha_1 (1 - \alpha_2) \alpha_3\)
  - \(p_{00} = (1 - \alpha_1) + \alpha_1 (1 - \alpha_2)(1 - \alpha_3)\)

![Diagram of a simple tree with labeled nodes and edges]
Performance Tomography in a Simple Tree

• Measurement
  – Dispatch \( n \) independent multicast probes from source \( 0 \)
  – Record outcomes for each packet, each leaf (received or not)

• Compute measured outcome frequencies
  – For each \( (ab) \in \{ (1,1), (1,0), (0,1), (0,0) \} \)
  – \( p'_{ab} \) = fraction of probes for which outcome is \( ab \)

\[ \begin{align*}
\text{Source} & \quad \alpha_1 \quad \alpha_2 \quad \alpha_3 \\
\text{Receivers} & \quad 2 \quad 3
\end{align*} \]
Performance Tomography in a Simple Tree

- Estimation
- Equate measured frequencies $p'$ with theoretical counterparts $p$
  \[ p'_{11} = p_{11} = \alpha_1 \alpha_2 \alpha_3 ; \]
  \[ p'_{10} = p_{10} = \alpha_1 \alpha_2 (1 - \alpha_3) ; \]
  \[ p'_{01} = p_{01} = \alpha_1 (1 - \alpha_2) \alpha_3 \]

- 3 independent equations, 3 unknowns $\alpha_1$, $\alpha_2$, $\alpha_3$
- 4th equation $p'_{00} = p_{00}$ not independent: frequencies sum to 1
- Solution
  - Estimates $\alpha'_1$, $\alpha'_2$, $\alpha'_3$ as solutions to equations

\[ \alpha'_1 = \frac{(p'_{11} + p'_{10})(p'_{11} + p'_{01})}{p'_{11}}, \alpha'_2 = \frac{p'_{11}}{p'_{11} + p'_{01}}, \alpha'_3 = \frac{p'_{11}}{p'_{11} + p'_{10}} \]
General Loss Estimator and Properties

• Leaf descendent sets
  - \( R(k) = \{ \text{nodes } j \in R : j \text{ is descended from } k \} \)
• Define \( \gamma_k = \Pr_\alpha[ \text{probe reaches any } j \in R(k) ] \)
• Establish 1-1 relationship between
  - Link probabilities \( \alpha = \{ \alpha_k : k \in V \} \)
  - Leaf set probabilities \( \gamma = \{ \gamma_k : k \in V \} \)
• Formally write \( \gamma = G(\alpha) \)
• Theorem
  • \( G \) is 1-1 with differentiable inverse
General Loss Estimator and Properties

- Measurement
  - Dispatch $n$ independent multicast probes from source 0
  - Record outcomes for each packet, each leaf (received or not)
  - For all $k \in V$, compute $\gamma'_k$
    - Fraction of probes that reach any $j \in R(k)$

- Estimation: $\alpha' = G^{-1}(\gamma')$
General Loss Estimator and Properties

• Theorem
  – $G$ is 1-1 with differentiable inverse $G^{-1}$

• Details
  – set $A_k = \text{Prob}[\text{packet reaches } k \text{ from } 0]$
    $\quad = \prod_{j \text{ ancestor of } k} \alpha_k$
  – can show $(1 - \gamma_k / A_k) = \prod_{j \in d(k)} (1 - \gamma_j / A_k)$
    • $d(k) = \text{children of } k$
  – $A_k$ is root of polynomial of degree $|d(k) - 1|$
    • with coefficients involving $\gamma_k$, and $\{\gamma_j : j \in d(k)\}$

• Find $\alpha_k$ as quotients of $A_k / A_{f(k)}$
  – $f(k) = \text{parent of } k$
General Loss Estimator and Properties

- Proof that \((1 - \gamma_k / A_k) = \prod_{j \in d(k)} (1 - \gamma_j / A_k)\)
- \(\gamma_k / A_k = \text{Prob}[\text{reaches } R(k) \mid \text{reach } k]\)
- \(1 - \gamma_k / A_k = \text{Prob}[\text{don’t reach } R(k) \mid \text{reach } k] = \prod_{j \in d(k)} \text{Prob}[\text{don’t reach } R(J) \mid \text{reach } k] = \prod_{j \in d(k)} (1 - \gamma_j / A_k)\)
Statistical Properties of Loss Estimator

• Maximum Likelihood Estimator

\[ \alpha'_k = \arg\max_{\alpha} \Pr_{\alpha} \text{[measured data]} \]

• Strongly consistent (converges to true value)

\[ \alpha' \to \alpha \text{ as } \#\text{probes } n \to \infty \]

• Asymptotically normal

\[ \sqrt{n} \cdot (\alpha' - \alpha) \to \text{multivariate normal r.v. as } n \to \infty \]

\[ \text{Var}(\alpha'_k) \approx (1 - \alpha_k)/n \text{ for large } n, \text{ small loss } 1 - \alpha_k \]

– (MLE efficient [ minimum asymptotic variance])

• Model is identifiable

– distinct link parameters \( \alpha \) yield distinct leaf loss distributions