On Transfer Function Modeling of Price Responsive Demand: An Empirical Study

Jaeyong An,  \textit{Student Member, IEEE}, P. R. Kumar,  \textit{Fellow, IEEE}, and Le Xie, \textit{Member, IEEE}

\textbf{Abstract}—This paper poses the problem of modeling price responsive demand as one of identifying the transfer function between the price and power consumption. This is motivated by the team’s earlier work of econometric estimation of the self- and cross- elasticity of demand response. It is discovered that electricity consumption has several unique features that may render the traditional approach ineffective. Such features include (1) nonlinear response between moderate and extremely high prices; and (2) time delay associated with any response from high prices. Based on the realistic data obtained from commercial and industrial loads in Texas, a transfer function modeling of price responsive demand is proposed. The effectiveness of this modeling approach is critically assessed. It is suggested that a transfer function modeling of price responsive demand could potentially be a fruitful direction to understand and close the loop around demand response in the smart grid.

I. INTRODUCTION

\textsc{S}mart grid refers to a flexible and cost effective power delivery network between a diverse set of energy supplies and informed power consumers. Among the smart grid technologies, \textit{demand response} (DR) provides a key mechanism to extract flexibility from the informed consumers for flexibility.

Among many smart grid investments, one of the major activities around the world is the massive deployment of advanced metering infrastructure. The payback from this major investment on data infrastructure is anticipated to be (a) enhanced flexibility from demand response participation for smart aggregators; and (b) improved real-time situational awareness for the grid operators. While streaming data in the smart grid provides unprecedented opportunities to transform the grid operation, we note that most prior research in this area falls into two categories: (i) data-driven static analytics tailored for power system domain applications, which do not capture the underlying coupling between the data and the dynamics in complex human-physical power systems [1] [2]; and (ii) model-based system theoretical studies which are difficult to scale up for real-world testing [3] [4].

The operations of power systems have traditionally adopted the philosophy of controlling generation to balance the stochastic demand. As a result, the dynamic modeling and control of power systems have been primarily focusing on generator side. Governor-turbine-generator (GTG) modules from various fossil fuels are modeled from first principles, resulting in a mature modeling taxonomy with well engrained notions such as droop characteristics and ramp rate [5]. More recently, there has also been work on modeling renewable energy sources such as wind farms as stochastic dynamic systems controlled by doubly-fed induction generators [6]. During the era when the prevailing paradigm was that supply follows demand, this modeling of supply side was enough to develop a coherent resource allocation framework for power systems. However, with the advent of demand response where demand too can be viewed as a \textit{controllable} entity, it has become imperative to symmetrically develop models for analyzing demand response too as a dynamical system with well defined inputs and outputs. The goal of this paper is to develop just such a dynamic system viewpoint for the demand side.

The central contribution of this paper is to exhibit from analysis of anonymous commercial/industrial (C/I) data\textsuperscript{1} that it is indeed possible to model demand response to prices as such a dynamical input-output system. Our contributions based purely on analysis of empirical data are twofold:

1) The response to large prices (over 95%-quantile: $144.42) can be modeled as a \textit{Hammerstein system}, i.e., a static nonlinearity followed by a linear transfer function [7]. Such large prices rarely persist for longer than a quarter-hour duration, and so the demand response can be viewed as a response to a price spike of a specific amplitude. We show that the resulting demand response indeed appears to be an impulse response of a nonlinear transformation of the initiating large price. The nonlinear transformation captures the fact that the demand reduction is not proportionate or linear in the price swing initiating the demand response, i.e., a 100$\times$ price increase does not result in a reduction that is five times the response to a 20$\times$ price increase. After accounting for this nonlinear transformation, which is typically concave since the response is sublinear, the response exhibits a reduction after a delay of about 0.75-2.5 hours, before subsequently reverting back to normal levels. Fig. 3 shows a typical demand response gleaned from the nine months data (Jan.1 - Sep. 30, 2008) available to and analyzed by us.

2) The response to moderate prices (up to $144.42) can be modeled as a linear stochastic system, specifically as an \textit{autoregressive exogenous} (ARX) system, i.e., an autoregressive (AR) system with exogenous input and white noise.

The rest of this paper is organized as follows. In Section II, we provide an overview on the characteristic of load and price

\textsuperscript{1}Anonymous even to us.

---

This material is based upon work partially supported by NSF under Contract Nos. CPS-1239116, ECCS-1150944, CCF-1331863, and Science & Technology Center Grant CCF-0939370.

J. An, P. R. Kumar, and L. Xie are with Department of Electrical and Computer Engineering, Texas A&M University. College Station, TX 77843. Email: jyan@tamu.edu, prk@tamu.edu, lxie@ece.tamu.edu.
data by preliminary analysis. We describe a transfer function model on DR with a comprehensive assessment in Section III. Concluding remarks are presented in Section IV.

II. PRELIMINARY DATA ANALYSIS

In Fig. 1, the C/I load and prices from Electric Reliability Council of Texas (ERCOT) measured at intervals of 15 minutes from Jan. 1, 2008 to Sep. 30, 2008 is plotted with respect to time. Figs. 1(a) and 1(b) are presented as boxplots. A boxplot is a graphical approach of depicting groups of data through their quartiles. While the bottom and top of the box are the first and third quartiles, the length of each whisker is equal to 1.5 × interquartile range (IQR), i.e., the height of the box. The first point which can be easily observed here is that the plot on price (Fig. 1(b)) shows many outliers while the plot on load rarely has them. This shows the “spiky” nature of price series, an abrupt and irregular sudden extreme price change for a very short term of 15-30 minutes duration (Fig. 1(d)). This makes the price highly non-normally distributed with heavy tail. Such spiky nature of prices can be explained by either the high marginal cost of production by the generators with the ability to respond rapidly to meet peak demand (e.g., gas or oil fired plants), or the bidding or withholding strategy of utility companies to maximize their profit. The other notable feature we see in Fig. 1 is that the load time-series exhibits a depressed demand in the afternoons, over time intervals overlapping fairly well with the time intervals which show frequent large outliers in the price time series. We infer that the depressed demand is developed as a consequence of demand response, and that the demand response is highly related to the outliers of price, because the depression can be hardly explained by the plot of the median of prices (Fig. 1(c)).

In Fig. 2, the statistics of the prices (P) and C/I load (Q) on workdays are depicted. From Fig. 2(b), the empirical probability plot of load versus the normal distribution shown by the diagonal dashed line, we can see that the empirical distribution of the load is fairly close to the normal distribution. For further validation, we can also check an estimate of the kurtosis, $\mu_4/\sigma^4$, where $\mu_n$ is the n-th moment about the mean and $\sigma$ is the standard deviation. It is 2.77, which is close to the value 3.0 for the normal distribution. Also its skewness, $\mu_3/\sigma^3$, is 0.11, which is close to the value 0 for the normal distribution. (Table I). Therefore, we can conclude that the distribution of the load is very close to a normal distribution. We also see that the load shows a highly correlated structure with the past load in Fig. 2(c), the plot of autocorrelation (ACF) of the load, while the partial autocorrelation (PACF) of the load (Fig. 2(d)) decays rapidly not exceeding lag of five quarter hours (75 minutes). Taking these facts into account, it is highly likely that a simple autoregressive (AR) model of order 3 or 5 would sufficiently well describe the load process.

On the other hand, the first characteristic of price we can observe in Fig. 2(a) is that the distribution of prices is highly non-normal. At low to moderate prices, the cumulative distributions matches with the diagonal dashed line, suggesting closeness to the normal distribution. However, the top 5% of the prices severely deviate from the line, reflecting the spiky nature of electricity prices. Such a long-tail property yields huge kurtosis (149.0002) and skewness (10.9133) as presented.
in Table I.

From the above, it is clear that it is not feasible to get a linear relationship between load and price over all values of \( P \) and \( Q \). Hence, we conclude that it is not possible to obtain one universal linear dynamic system model between price and demand. As an alternative, it is natural to continue the analysis by assuming that there are two transfer functions (TFs), one for moderate prices which is a linear model, and one for high prices where there are large values. The deviation from normality of the top 5% in Fig. 2(a) gives a reasonably good demarcation between moderate prices and peak or high prices.

III. ESTIMATION OF DYNAMIC MODEL ON LOAD AND PRICE

From the aforementioned preliminary data analysis in Section II, we conjecture that there exist two qualitatively distinct regimes, a low to moderate price regime, and a high price regime. In the former we consider a linear transfer function between price and load with additional noise to account for uncertainty, i.e., an ARX model driven by white noise. In the high price regime we consider a concave transformation of peak prices to account for non-normality of the process. In this section, we further address this problem of identifying the dynamic model of DR.

A. Methodology

In this section, we briefly discuss the estimation and validation methodology to estimate the dynamic model of DR. For the basic dynamic model of DR, we consider an ARX model driven by white noise, one of the simplest but most useful models for forecasting and control. For estimation, we consider the least squares (LS) method for estimating the unknown parameters in a linear regression model [8], [9]. To verify the existence of DR and the significance of the results of estimated parameters, we use the analysis of variance (ANOVA) method [10]. For examining the minimum net contribution of price information to reduction of error in load estimation, we consider a two-step estimation procedure. To achieve parsimony of the model, we cross-validate the model by a random division of each complete data set under two separate conditions (i.e., moderate prices and high prices) into two sets of the same size, namely, a training set and a test set. We estimate the model from the training set and evaluate it on the test set.

1) Autoregressive Exogenous (ARX) model: Denote by \( \{ P(t) \}_{i=1}^N \) and \( \{ Q(t) \}_{i=1}^N \) the time series of prices and loads, each consisting of \( N \) observations. If we denote by \( z^{-1} \) the backshift operator \( z^{-1}X(t) := X(t-1) \), the ARX model can be described as follows:

\[
\alpha(z^{-1})Q(t) = \beta(z^{-1})P(t) + \epsilon_t, \tag{1}
\]

where \( \alpha := [1 - \alpha_1 - \alpha_2 \ldots - \alpha_m] \) and \( \beta := [\beta_1 \beta_2 \ldots \beta_n] \) are unknown parameters to be estimated, \( \alpha(z^{-1}) := [\alpha^t \cdot z^{-i}]_{i=1}^m \) and \( \beta(z^{-1}) := [\beta^t \cdot z^{-i}]_{i=1}^n \) are the characteristic and numerator polynomial of TF respectively, and \( \epsilon_t \) is an error which is an independent and identically distributed (i.i.d.) noise process with \( E\epsilon_t = 0 \) and \( \text{VAR}\epsilon_t = \sigma^2 \).

TABLE II. ESTIMATED AR MODEL OF \( Q(t) \)

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Estimate</th>
<th>SE</th>
<th>tStat</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>238.07</td>
<td>13.989</td>
<td>17.018</td>
<td>( 8.883 \times 10^{-64} )</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.81268</td>
<td>0.0085477</td>
<td>95.075</td>
<td>0</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>0.04606</td>
<td>0.010267</td>
<td>4.4886</td>
<td>( 7.2744 \times 10^{-6} )</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>0.036614</td>
<td>0.0085466</td>
<td>4.284</td>
<td>1.8579 \times 10^{-5}</td>
</tr>
<tr>
<td>( \sqrt{\text{MSE}} )</td>
<td>301</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic vs. constant model:</td>
<td>8.81 \times 10^3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.775</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Two-step Estimation: Our primary objective in this work is to show the existence of DR and understand it from a dynamic system perspective. We employ the following two-step estimation procedure to examine the net contribution of price information to reduction of error in load estimates.

1) First estimate the regression parameters \( \hat{\alpha} \) and obtain

\[
Q_{res}(t) := (1 - \sum_{i=1}^m \hat{\alpha}_i z^{-i})Q(t).
\]

2) Estimate \( \hat{\beta} \) using the equation

\[
Q_{res}(t) = (\sum_{i=1}^n \hat{\beta}_i z^{-i})P(t) + \epsilon_t.
\]

Then, the overall estimated ARX model is the following:

\[
Q(t) = (\sum_{i=1}^m \hat{\alpha}_i z^{-i})Q(t) + (\sum_{i=1}^n \hat{\beta}_i z^{-i})P(t) + \epsilon_t, \tag{2}
\]

where \( \hat{\alpha}_i \) and \( \hat{\beta}_i \) are the LS estimators of \( \alpha_i \) and \( \beta_i \).

B. Demand Response to Moderate Price

In this section, we present an ARX model for DR in the moderate price regime, the prices below the 95%-quantile. Tables IV shows the overall estimation results of the ARX model. The estimated TF of the model is:

\[
TF_{\text{Low}} = \frac{-0.8555z^{-1} + 0.5273z^{-2}}{1 - 0.8127z^{-1} - 0.0461z^{-3} - 0.0366z^{-5}}. \tag{3}
\]

This model explains 77.6% of the variance that \( Q(t) \) initially possesses. Tables II and III present the results of the analysis for each of the two steps of estimation. The Estimate column shows the estimated coefficient value, the SE refer to the standard error of the estimate, the tStat indicates the t-statistic for a hypothesis test that the coefficient is zero, and the pValue is the p-value for the t-statistic.

What we see here is that though price has sufficient statistical significance due to its low p-value (0.0147), its innovative contribution to the load forecast is relatively small (less than 0.1%), and most of the change in \( Q(t) \) can be explained by the past of the load itself (AR(5) model). This suggests that a moderate price has very little impact in eliciting demand response, which is also consistent with our observation in the preliminary analysis in Section II.

C. Demand Response to High Price

In this subsection, we present an ARX model for the high price regime, where the prices are over the 95%-quantile (144.4187 $/MWh). A sample time-series of a typical load evolution after a high price spike is shown in Fig. 3(a). Here, we observe a huge drop of the load after a one and half hour lag. Fig. 3(b) shows that such a load drop phenomenon is
not an isolated event; we generally see such a general load drop and recovery pattern over two and half hours after price surges. The ANOVA result in Fig. 3(b) in Table V supports our observation that there exists a significant load drop 0.5-1.5 hours after a price surge due to its extremely low p-value ($3.86 \times 10^{-4}$). This is sufficiently low to reject the null hypothesis of a constant model for the load.

In addition, we also observe that the height of price surge is correlated to the depth of load drop from Fig. 3(c) and 3(d). Fig. 3(c) shows the average curve of the change $Q(k)$, at a certain level of price surge $P$ at time $t$, where $Q(k) = \frac{1}{|P|} \sum_{t \in P} Q(t+k) - Q(t)$ for all $P$ in a subset of sample prices $P = \{P(t) : P_{min} \leq P(t) \leq P_{max}\}$ for given $P_{min}$ and $P_{max}$. We see that higher $P_{min}$ and $P_{max}$ result in the greater load drop. Fig. 3(d) shows the correlation between the height of the price surge ($\Delta P = P(t) - P(t-1)$) and the load $Q$, which is most negatively significant after $k = 5$ quarter-hour periods (i.e., one hour and 15 minutes) from a price surge.

Based on the above observations, we establish a simple dynamic model between the magnitude of the price surge and the load, for high price surges. Taking into account the long-tailed characteristic of prices, we consider a linear model in the convex transformation $\log P(t)$, instead of $P(t)$, for better estimation performance. Moreover, because of the innate time-dependency on DR, we present a TF for a specific time period, from 2:00pm to 2:30pm, in this paper. The estimation results for the ARX model of DR at high price are shown in Tables VI, VII, and VIII. The estimated TF of the ARX model is:

$$T_F^{10\text{pm}} = \frac{-220.1z^{-4}}{1 - 0.4015z^{-1} + 0.2383z^{-2} - 0.2512z^{-4}},$$

which explains 51.2% of the variance that $Q(t)$ has. The first point we observe here is that the accuracy of the AR model for $Q(t)$ is severely degraded ($R^2 = 33.2\%$) in Table VI, compared to the AR model for the moderate price regime (Table II). However, we see that a relatively high portion (27%) of the variance of $Q_{res}(t)$ is explained by the estimated model of $Q_{res}(t)$ shown in Table VII, from which we conclude that the innovation from the price information is significant to improve $R^2$ of ARX model up to 51.2% as shown in Table VIII.

In Fig. 4, we check the validity of our model by sample load forecast. Figs. 4(a) and 4(b) depict the errors in the load forecast at 3:15pm after a price surge at 2:15pm. We see that the forecasted $\hat{Q}(t)$ and the actual $Q(t)$ at $t = 3:15$pm are fairly well correlated (correlation $r_{Q\hat{Q}} = 0.7160$) in Fig. 4(b), and that the errors exhibit normality (Kurtosis = 3.1809) in Fig.

---

**TABLE III. ESTIMATED LINEAR MODEL OF $Q_{res}(t)$**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>SE</th>
<th>tStat</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>22.506</td>
<td>10.054</td>
<td>2.2385</td>
<td>0.025218</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.8555</td>
<td>0.42677</td>
<td>-2.0046</td>
<td>0.045043</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.5273</td>
<td>0.43006</td>
<td>1.2261</td>
<td>0.2202</td>
</tr>
</tbody>
</table>

$\sqrt{\text{MSE}}: 301$ $R^2: 0.0084$ F-statistic vs. constant model: 4.22 p-value = 0.0147

**TABLE IV. THE ARX MODEL ON $Q(t)$**

\[
(1 - \alpha_1z^{-1} - \alpha_3z^{-3} - \alpha_5z^{-5})Q(t) = (\beta_1z^{-1} + \beta_2z^{-2})P(t) + \epsilon_t + \epsilon_0
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Coefficient</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.81268</td>
<td>$\beta_1$</td>
<td>-0.8555</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.046086</td>
<td>$\beta_2$</td>
<td>0.5273</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.036614</td>
<td>$\epsilon_0$</td>
<td>260.126</td>
</tr>
</tbody>
</table>

$\sqrt{\text{MSE}}: 301$ $R^2: 0.776$

**TABLE V. ANOVA RESULTS FOR FIG. 3(b)**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>$F$</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>$1.21 \times 10^7$</td>
<td>10</td>
<td>$1.21 \times 10^6$</td>
<td>3.21</td>
<td>$3.86 \times 10^{-4}$</td>
</tr>
<tr>
<td>Error</td>
<td>$3.89 \times 10^9$</td>
<td>10351</td>
<td>$3.76 \times 10^5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$3.90 \times 10^9$</td>
<td>10361</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SS: Sum of squares; DF: Degree of freedom of error; MS: Mean square; F: F-statistic.

---

**TABLE VI. ESTIMATED AR MODEL FOR $Q(t)$**

\[
Q(t) = \alpha_1 Q(t-1) + \alpha_2 Q(t-2) + \alpha_4 Q(t-4) + \alpha_6 + Q_{res}(t)
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>SE</th>
<th>tStat</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>748.26</td>
<td>233.72</td>
<td>3.2015</td>
<td>0.0025097</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.40153</td>
<td>0.11763</td>
<td>3.4133</td>
<td>0.0013678</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.23826</td>
<td>0.1461</td>
<td>-1.6308</td>
<td>0.09992</td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>0.25124</td>
<td>0.11516</td>
<td>2.1816</td>
<td>0.0344</td>
</tr>
</tbody>
</table>

$\sqrt{\text{MSE}}: 336$ $R^2: 0.332$ F-statistic vs. constant model: 7.44 p-value = 0.000377

**TABLE VII. ESTIMATED LINEAR MODEL FOR $Q_{res}(t)$**

\[
Q_{res}(t) = \beta_4 P(t-4) + \beta_0 + \epsilon_t
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>SE</th>
<th>tStat</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>1213.4</td>
<td>293.68</td>
<td>4.1316</td>
<td>0.00014688</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-220.1</td>
<td>52.774</td>
<td>-4.1707</td>
<td>0.00012965</td>
</tr>
</tbody>
</table>

$\sqrt{\text{MSE}}: 281$ $R^2: 0.27$ F-statistic vs. constant model: 17.4 p-value = 0.00013
TABLE VIII. THE ARX MODEL FOR $Q(t)$

\[
(1 - \alpha_1 z^{-1} - \alpha_2 z^{-2} - \alpha_4 z^{-4})Q(t) = \beta_4 z^{-4} \log P(t) + \epsilon_t + \epsilon_0
\]

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Estimate</th>
<th>Coef.</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.49153</td>
<td>$\beta_4$</td>
<td>-220.1</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.23826</td>
<td>$\epsilon_0$</td>
<td>1961.66</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.25124</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\sqrt{\text{MSE}} = 281$ $R^2 = 0.5124$

(a) The probability plot of $\epsilon$ for normal distribution (Kurtosis = 3.1809). (b) The plot of $\hat{Q}$ over $Q$ ($r_{QQ} = 0.7160$).

Fig. 4. The plots of prediction error $\epsilon$.

4(a).

In Fig. 5, we investigate the time dependency of the ARX model for a high price surge. Fig. 5(a) shows that the time lag in the TF has some randomness, ranging from 0.75 hours to 2.75 hours. Fig. 5(d) suggests that the period of the day in which DR demonstrates statistical significance is from 1:15pm to 2:75 hours. Fig. 5(c) and 5(d)), for which the TF is shown in Equation (4).

IV. CONCLUSION

This paper poses the problem of modeling of price responsive demand at wholesale level. Based on the empirical data acquired at ERCOT, we propose a dynamical transfer function perspective of modeling such behavior. Empirical study suggests that (1) the price responsiveness of demand may have qualitatively different behavior during “normal price” and “peak price” periods; and (2) there exists a demand response delay consequent on a high price surge. The first suggestion can be reasonably interpreted as saying that there is little incentive for increasing power consumption at low price since we do not have efficient energy storage yet. On the other hand, this suggestion is in line with the observation in financial markets that a financial market tends to react more sensitively to bad news than good news [11]. The second suggestion shows that there exists a certain “inertia” in consumption so that it takes a certain time delay to reduce power consumption after a peak price observation.

Perhaps more important than the two specific findings is the potential value of the very approach of employing transfer functions for flexible demand modeling. This modeling approach offers many more degrees of freedom in characterizing the salient nature of power consumers as compared with classical econometric modeling of price elasticity. This work is but a first step toward modeling and closing the loop around price responsive demand data. Future avenues of research include design of optimal pricing, cross-validation of such a modeling approach in various regions, and a probabilistic characterization of demand response with large aggregated customers.

ACKNOWLEDGMENT

The authors would like to acknowledge the data provided by Professor S. Puller on price responsive demand.

REFERENCES